The Formation of Water, Glycerine and Ethyl Alcohol Barometers

Ajay Sharma*

Abstract

It is correctly established that g decreases with altitude, but the variation of g with atmospheric pressure (decreases with altitude) is not considered in precise experiments in the existing literature. Torricelli determined in pioneering experiments in 1644 that the height of a mercury column in a barometer was 0.76 m due to atmospheric pressure. Newton formulated g in 1685, and then Pascal's Law was treated in the presence of gravity for an imaginary cylinder of liquid. Thus the equation $P = dgH$ is obtained which relates acceleration due to gravity, g, with atmospheric pressure, P. The expression for variation in g with altitude as $g_h = g/(1 + h/R)^2$ by both methods will be compared. At sea level the heights of liquid columns (for water 10.33 m, glycerine 8.202 m, ethyl alcohol 13.16 m) are independent of other factors such as diameters of tubes, viscosity, surface tension of liquid, angle of contact and capillarity, etc. At height of 2 km above the surface of the earth the heights of liquid columns are reduced, *e.g.* for mercury 0.5967 m, water 8.1158 m and glycerine 6.4411 m. Now measuring P, H and g can be calculated. The value of g can be determined by both methods at various heights and should be the same. Theoretically when atmospheric pressure becomes zero then the value of g_h (P/dH) must tend to zero; according to $g_h = g/(1 + h/R)^2$, g_h becomes zero at infinite large distances. But no such attempts have been reported in literature, hence it is an open problem, especially when tubes of various diameters are considered and characteristics of liquids are different. Due to diverse experimental conditions of liquids and equipments, mercury may be regarded as an ideal liquid for such measurements of pressure. The value of g due to altitude decreases steadily, whereas due to atmospheric pressure g decreases abruptly. So sensitive experiments are absolutely necessary to draw concrete conclusions.

Introduction

Torricelli formed a mercury barometer in 1644 and confirmed the height of a mercury column as 0.76 m and acceleration due to gravity was defined by Newton in 1685, then the expression for pressure[1-3] was determined as

$$P = dgH \tag{1}$$

and was calculated using Pascal's Law in the presence of gravity. Here P is pressure; g is acceleration due to gravity and H is height of liquid column. Equation 1 is derived for an imaginary cylinder of liquid. Consider a liquid of density d is in equilibrium of rest. Therefore net force acting on the cylinder will also be zero, *i.e.*

$$F_1 + Mg - F_2 = 0 \tag{2}$$

where F_1 is force acting vertically downwards on the top face of the cylinder, F_2 is the force acting vertically upwards on the lower face of cylinder and Mg is the weight of mass of water in the cylinder.

$$P_1A + AHdg - P_2A = 0 \tag{3}$$

where P_1 is pressure at the upper end, P_2 is pressure at the lower face and A is the area of upper and lower surfaces.

Pressure exerted by liquid column

$$P_2 - P_1 = \mathrm{P} \tag{4}$$

or $P = dgH$ (Equation 1). The unit of pressure, Pascal, was defined by simply putting value of H equal to 0.76 m. So, Equation 1 becomes

$$P = 13{,}600\ \mathrm{kgm^{-3}} \times 9.8\ \mathrm{ms^{-2}} \times 0.76\ \mathrm{m} = 1.013\ 10^5\ \mathrm{Pa} \tag{5}$$

This may be regarded as the standard equation for atmospheric pressure. In Equation 1, H is the height of an imaginary cylinder of liquid which is submerged in the same liquid (the same liquid is above and below the cylinder). Whereas in the case of a mercury barometer, the height H of liquid column is 0.76 m, and above the mercury column there is vacuum. Whereas while deriving $P = dgH$, the same liquid is considered above and below the imaginary cylinder of liquid.

Further, according to Newton's law of gravitation, the acceleration due to gravity decreases as altitude increases. The acceleration due to gravity at the surface of the earth

$$g = GM/R^2 \tag{6}$$

where G is gravitational constant, M is mass of the earth and R is radius of the earth. Acceleration due to gravity g_h at height h is

$$g_h = GM/(R + \mathrm{h})^2 \tag{7}$$

From Equations 6 and 7,

$$g_h = g/(1 + h/R)^2 \tag{8}$$

The acceleration due to gravity at height h is given by Equations 7 and 8 and acceleration due to gravity by method of variation of pressure is given by

$$g_H = P/dH \quad (9)$$

Both Equations 8 and 9 give value of g, but by entirely different methods.

Comparison for Acceleration Due to Gravity by Both Equations: $g_H = P/dH$ and $g_h = g/(1+h/R)^2$

At different heights acceleration due to gravity is shown by both equations, *i.e.* $g_h = P/dH$ and $g_h = g/(1+h/R)^2$. In one equation g_h depends upon P, d and H, whereas in the other it only depends upon altitude, h. According to $g_h = g/(1+h/R)^2$ the value of g decreases steadily whereas according to $g_h = P/dH$, the same decreases abruptly.

The measurement of g due to "method of atmospheric pressure" and "variation with altitude" implies that g decreases with height, h. According to Equation 8, g depends upon height h. Whereas in Equation 1 or 9, g indirectly depends upon height h as pressure decreases causing decrease in height of the liquid column H. At height of 50 km the total air is only 1% implying considerable decrease in pressure as atmospheric pressure decreases. At height of 50 km, atmospheric pressure is 75.944 Pa and the same at height of 25 km is 2511.02 Pa. Thus accordingly g will decrease.

1) The variables in Equation 1 are density of liquid which can be kept constant: pressure P can be measured by various methods, height of the column H has to be directly measured. There are no such factors like diameter of tube (tubes can be of different diameters), viscosity, surface tension, capillarity and angles of contact, etc. Equation 1 is valid for all liquids and tubes. The various characteristics of liquids are shown in Table 1.

1a) For a mercury barometer, the height of liquid column is 0.76 m. It is clear from Equation 5 the height of the mercury column is regarded as standard and gives unit of pressure 1.013×10^5 Pa. This magnitude of pressure may be used for calculating the height of liquid columns of various liquids.

1b) The heights of liquid columns of ethyl alcohol, water and glycerine barometers are given by 13.16 m, 10.33 m and 8.2 m at sea level. In spite of availability of precise measurement techniques, such experiments are not conducted yet.

2) The acceleration due to gravity also varies with altitude, and theoretically tends to zero at infinity, according to equation $g_h = g/(1+h/R)^2$. Also according to $g_H = P/dH$, acceleration due to gravity becomes zero when $P = 0$ provided d and H are finite. At higher altitude the pressure decreases abruptly, hence g_H.

3) The various typical predictions for heights of liquid columns are shown in Table 2. At distance of 2 km the heights of mercury, water and glycerine columns are 0.5969 m, 8.1158 m and 6.4411 m. These values are lower than values at sea level and higher than values at 8 km (typical height of peaks of Mount Everest). For completeness the values H are determined at heights 25 km (2511.02 Pa) and 50 km (75.944 Pa) above sea level.

3a) Acceleration due to gravity is more at poles than at the equator, hence accordingly the height of the liquid column will vary accordingly.

Critical Discussion

1) According to equation $P = dgH$, the pressure is completely independent of other factors such as diameters of tubes, viscosity, surface tension of liquid, angle of contact and capillarity, etc.

There is no factor which takes into account the diameter of the tube in which the height of the liquid column is measured. Theoretically, the height of the liquid column must be the same for the capillary tube (closed upper end) and tube of diameter 2 feet. However, the phenomena of rise or fall of liquids is observed in capillary, whereas the upper end is open. This aspect

Table 1. Comparison of various characteristics of ethyl alcohol, water, glycerine and mercury.

Characteristic	Ethyl Alcohol	Water	Glycerine	Mercury
Density (kg/m^3)	785	1,000	1260	13600
Coeff. of viscosity (poise)	1.32×10^{-4}	1.01×10^{-2}	10.69	15.5×10^{-3}
Surface tension(dyne/cm)	22.3	75.6	63.1	465 (dyne/cm)
Angle of contact	<90°	8-9^0	--	137°
Capillarity	Rise	Rise	---	Fall
Physical behavior	Wet	Wet	Wet	Does not Wet
Height liquid column (h) in m	13.16	10.33	8.202	0.76

Table 2. Heights of liquid columns of different liquids in tubes of different diameters, at different heights.

Sr. No	Height (m)	Pressure (Pa)	Acceleration (m/s^2)	Mercury $H = P/dg$	Water $H_1 = P/d_1g$	Glycerine $H_2 = P/d_2g$
1	0	101300	9.8	0.76	10.33	8.202
2	1	89,874.57	9.7969	0.6743	9.1737	7.2807
3	2	79,485.2	9.7938	0.5967	8.1158	6.4411
4	3	70,108.5	9.7908	0.5265	7.1606	5.683
5	4	61,640.2	9.7877	0.4630	6.2977	4.9981
6	5	54,019.9	9.7847	0.4059	5.5208	4.3816
7	6	47,181	9.7816	0.3546	4.8234	3.8281
8	7	41,060.74	9.7785	0.3087	4.1990	3.3326
9	8	35,599.8	9.7755	0.2677	3.6417	2.8902
10	25	2511.02	9.6486	0.01915	0.26047	0.20672
11	50	75.944	9.50078	0.00058	0.007999	0.006252

is not taken into account in equation $P = dgH$.
2) Mercury does not wet the walls of the container, whereas water does. This property is likely to affect the height of the water column.
3) Similarly, glycerine is the most viscous liquid; its viscosity is 689.67 times that of mercury and 1,058 times that of water. It causes internal resistance of the fluid and retards motion; it may affect the stabilization of liquid column. The viscous fluids also wet the tube.
4) The acceleration due to gravity at any height ($g_H = P/dH$) can be calculated by measuring atmospheric pressure P and height of liquid column, H. At any height the atmospheric pressure P can be known from standard atmospheric calculator and H has to be experimentally confirmed for various liquids (mercury, water, glycerine, etc). Thus g_H can be measured.

On the other hand acceleration due to gravity $g_h = g/(1+h/R)^2$ varies with height which can be easily estimated. Now both values of g must be the same. For consistency of both values of g, the heights of liquid columns are shown in Table 2.
5) Thus undoubtedly mercury may be regarded as an ideal liquid for such measurements as it has low viscosity, hence comes to rest quickly, and does not wet the walls of the glass tube. But the experiments have to be conducted under diverse conditions for various liquids, in view of Equation 1 for general conclusions.

Conclusions

The acceleration due to gravity, g, varies with altitude and it can be measured with two methods, *i.e.* $g_H = P/dH$ and $g_h = g/(1+h/R)^2$. The values of g_h by both methods must be the same. The acceleration due to gravity has not yet been measured by the former method. Moreover, in this case there are many parameters for which the equation can be tested, and only then can general conclusions be drawn. For consistency of both the methods, the values of height of liquid columns (for mercury, water and glycerine) are 0.6743 m, 9.1737 m and 7.2807 m at height of 2 km above sea level. Such observations have not been taken by scientists in the existing literature. In this regard it can be added that equation $g_h = P/dH$ was derived for an imaginary cylinder of liquid which is submerged in the liquid of the same density. Whereas in determination of height of liquid column, above the liquid column there is vacuum. Further, apparently some significant factors such as diameters of tubes, viscosity, surface tension of liquid, angle of contact and capillarity, etc. have been excluded. Also the equation was derived for an imaginary cylinder in equilibrium with liquid at rest. However, it is applied for practical purposes as cited above. Such factors may affect the results. Thus sensitive experiments are required to compare both values of g. It is equally possible, in view of diverse parameters of various liquids, that mercury has to be regarded as standard in measurements of pressure. The value of g due to altitude decreases steadily, whereas due to atmospheric pressure g decreases abruptly. So sensitive experiments are absolutely necessary to draw concrete conclusions.

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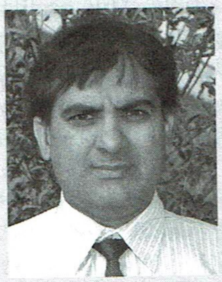

About the Author
Ajay Sharma has a Master's in Physics. While in school he began critically analyzing Aristotle's assertion and Galileo's views on falling bodies. He began his career as a physics lecturer at DAV College Chandigarh (India). He has authored three books on physics devoted to alternate theories, and a biography about his own interactions with the scientific community. He has published over 30 scientific articles on the basic laws. His work is available online at: http://www.AjayOnLine.us

*Fundamental Physical Society, His Mercy Enclave, P.O. Box 107, GPO Shimla 171001 HP India
Email: ajay.pqrs@gmail.com Newton99.Com