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ELASTIC COLLISIONS IN ONE DIMENSION AT MACROSCOPIC LEVEL AND NEWTON'S THIRD LAW OF MOTION

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In the existing literature the third law of motion (action and reaction are always equal and opposite) has been qualitatively studied in various cases. It can be quantitatively studied in case of recoil of gun comparatively easily and with reasonable accuracy. There is no single specifically assigned or unanimously agreed physical quantity which is used to denote or represent or express action and reaction. These are expressed in terms of velocity, momentum, force, push or pull depending upon the situation. In case of one dimensional elastic collisions at macroscopic level the third law of motion is not always true (both in magnitude, and direction). The equality of action and reaction depends upon various characteristics (nature, magnitude, shape, angle and velocities of colliding particles, roughness of surface etc.) of system. Depending upon the genuine and logical deduction, in case of elastic collisions the third law of motion has been re-stated or generalised as "to every action there may be a reaction but not always equal and opposite to action".

INTRODUCTION : When two bodies interact with each other one of two forces is termed as action and the other reaction. There is no cause-effect relation between two. Both occur simultaneously either force may be considered as action and other reaction [1].

There is no single specifically assigned or unanimously agreed physical quantity which is used to denote or regarded as a measure of action and reaction. So, they are expressed in terms of velocity, momentum, force, push or pull depending upon the situation. The relation between two is known as Newton's third law of motion as "To every action there is always equal and opposite reaction".

i.e. $$\text{Action} = -\text{Reaction} \quad ...(1)$$

There are many examples in daily life which support it qualitatively e.g. man swims in water. The man pushes water backward (action) and water thus pushes man in forward direction (reaction).

Backward push on water (action) = Forward push on man (reaction). When ball rebounds from target Forward velocity of ball (action) = Backward velocity (reaction) And in recoil of gun Forward momentum of bullet (mv) = Backward momentum of gun (MV) (2) In case of rocket propulsion as smoke and ejected gases move backward (action) from the rear of rocket; then rocket (fuel, tank and satellite) moves forward (reaction). At the same time in cases of helicopters and aeroplanes such gases are not ejected as in case of rocket, even then they move forward. Anyhow in case of rocket it is almost impossible to precisely measure the velocity and amount of gases ejected backward; and hence to draw conclusions about action and reaction quantitatively.

Among the many existing qualitatively examples supporting the third law of motion, it may be comparatively easier in case of recoil of gun to draw conclusions quantitatively after designing experimental set-up purposely. Then concrete conclusions can be drawn that under which conditions and upto which extent the Eq. (2) *i.e.* action and reaction law is obeyed quantitatively.

Here our primary aim is to discuss theoretically and critically the third law of motion in case of one dimensional elastic collisions at macroscopic level which involves conservation laws of

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momentum and kinetic energy simultaneously. In existing literature, in case of elastic collisions the action and reaction are studied in terms of initial and final velocities of colliding bodies. And mainly our approach is to study action and reaction of projectile. Also simply as a reference it may be pertinent to mention that already author has theoretically described a feasible/viable set-up to confirm mathematical predictions quantitatively in case of elastic collisions (co-efficient of restitution $e = 1$) at macroscopic level in one dimension [2].

A CRITICAL ANALYSIS OF THE THIRD LAW OF MOTION IN VIEW OF ONE DIMENSIONAL ELASTIC COLLISIONS : Let u and v be the initial and final velocities of the projectile and target. Then the both collide and their final velocity U and V are related with their masses m and M as (1, 3).

$$U = \{2Mv + u(m - M)\}/[m + M] \quad \ldots(3)$$

$$V = \{2mu + v(M - m)\}/[m + M] \quad \ldots(4)$$

Now considering for simplicity that the target is at rest *i.e.* $v = 0$, then

$$U = u[m - M]/[m + M] \quad \ldots(5)$$

$$V = 2mu/[m + M] \quad \ldots(6)$$

Now depending upon masses of projectile following three cases are possible.

(i) When mass of projectile (m) is negligible compared to that of target (M)

$$m - M \simeq -M \quad \text{and} \quad m + M \simeq M$$

Then Eq. (5) becomes

$$u \simeq -U \quad \ldots(7)$$

and $$V = 2mu/M \simeq 0$$

Initial velocity of projectile (Action)

$\simeq -$ Final velocity of projectile (Reaction) ...(7)

which is true as far as third law of motion is concerned. It represents elastic collision as co-efficient of restitution $[e = (V - U)/u = 1]$ is unity. But its precise practical realisation in sensitive experiments may not be so easy. The experimental results will directly depend upon nature, velocity, shape, composition of colliding particles, roughness of surface on which collision is accomplished and in addition to these the points (edge or centre) at which projectile hits the target.

The above fact is also supported by other observations in daily life *e.g.* if different balls (cricket, tennis, table tennis, ordinary ball and a specially fabricated ball) are hit on the wall or heavy lead target. Then only specially fabricated ball will rebound with same velocity and to the same point. Thus Eq. (7) will be experimentally confirmed under some conditions only; and such a system may be called an ideal system. It can be further justified theoretically as below.

(a) If $M = 2m$, then Eq. (5) becomes

$$u = -3U \quad \ldots(8)$$

$$V = 0.6667u$$

$$\text{Action} = -3 \text{ Reaction.}$$

$$e = (V - U)/u = 1$$

Thus in this case for projectile action and reaction are opposite but not equal, as reaction is one third of action.

(b) Also if $M = 500m$, then Eq. (5) becomes

$$u = -1.004U \quad \ldots(9)$$

$$V = 0.004u$$

$$\text{Action} = -1.004 \text{ Reaction,}$$

$$e = (V - U)/u = 1$$

Hence reaction only approaches to action in magnitude if mass of target is very-2 large compared to that of projectile.

(ii) When mass of target (M) is negligible compared to that of projectile *i.e.* $m + M \simeq m$ and $m - M \simeq m$ Now Eq. (5) becomes

$$u - U \qquad \text{...(10)}$$

$$V\ 2mu/m - 2u$$

$$\text{Action} \simeq \text{Reaction}$$

$$e = 1$$

Thus in this case action is equal or approaches to reaction in magnitude. But in this case projectile continues to move in the same direction *i.e.* action and reaction are not opposite. Hence the third law of motion is not completely justified (*i.e.* both in magnitude and direction) in this case. Also in this case action is equal to reaction (in magnitude) *i.e.* half or partial justification of third law of motion is even true only when M is negligible compared to m. If $m = 5M$, then Action = 1.5 Reaction. Also, Action = 1.0004 Reaction if $m = 5000\,M$ (in both cases $e = 1$). Hence, even partial or half justification or obeyance of third law of motion is under only special conditions.

(iii) Finally, let us consider the case if m and M are nearly or precisely equal.

(a) Let mass of projectile m is fractionally smaller than target $m = 0.9999M$, then

$$U = -0.00005u \qquad \text{...(11)}$$

$$V = 0.99995u$$

$$\text{Action } (u) \neq 19999 \text{ Reaction } (U) \qquad \text{...(11)}$$

Thus, in this case

$$\text{Action} \neq \text{Reaction (in magnitude)}$$

$$\text{Action} = -\text{Reaction (in direction)}$$

Thus in this case third law of motion is only justified as far as direction is concerned.

(b) Let mass of projectile is fractionally greater than that of target, $m = 1.0001M$.

$$U = 0.00005u \qquad \text{...(12)}$$

$$V = 1.00005u$$

and

$$\text{Action } (u) = 20001 \text{ Reaction } (U)$$

Thus in this case reaction is neither equal to action in magnitude nor in direction.

(c) If masses of projectile and target *i.e.* $m = M$

$$U = u.0 = 0 \qquad \text{...(13)}$$

$$V = u$$

Thus in this case for definite action (u), there is no reaction. Experimentally it can be easily observed if one marble properly hits the other in typical collision; then one marble (projectile) comes to rest and target starts moving ($V = u$). In all above cases co-efficient of restitution e is unity *i.e.* all these are perfectly elastic collisions as shown in Table 1.

Table 1 : The relationship between mass of projectile m, mass of target M, initial velocity of projectile u, final velocity of projectile U, initial velocity of target v and final velocity of target V in perfectly elastic collisions *i.e.* co-efficient of restitution e $\{(V - U)/u\}$ is unity.

	m	M	u	v	U	V	e
(i)	$m << M$	$M >> m$	u	0	$-u$	0	1
(ii)	m	$2m$	u	0	$-0.3333u$	$0.6667u$	1
(iii)	m	$500m$	u	0	$-0.996u$	$0.004u$	1
(iv)	$m >> M$	$M << m$	u	0	u	$2u$	1
(v)	$5M$	M	u	0	$0.6666u$	$1.6666u$	1
(vi)	$5000M$	M	u	0	$0.9996u$	$1.9996u$	1
(vii)	$0.9999M$	M	u	0	$-0.00005u$	$0.99995u$	1
(viii)	$1.0001M$	M	u	0	$0.00005u$	$1.00005u$	1
(ix)	m	m	u	0	0	u	1

Also in the existing literature [4] these equations are regarded as valid in atomic scale as well. The Eqs. (11–13) represent in broadway the category of collisions in which collision between fission neutron and protons in nuclear reactions can be analysed. Theoretically if we speculate head on collision with fission neutrons (200MeV) and a free proton in nuclear reactor then neutron should immediately lose its most of energy (thus becomes thermal neutron 0.025MeV) and proton should move with velocity comparable to that of neutron. But such speeding proton (speculated) may be undesired in reaction, however this process can be utilised in other cases. Consequently extent of validity of elastic collisions can be further checked in such cases. whereas Eqs. (7, 9) may be regarded as to represent the category of collisions, in which collisions between neutrons and heavy uranium atoms can be studied. Also similar examples may be collisions between alpha particles and heavy gold nuclei in Rutherford's scattering experiment.

THE GENERALISATION OF NEWTON'S THIRD LAW OF MOTION IN ONE DIMENSIONAL ELASTIC COLLISIONS AT MACROSCOPIC LEVEL : In case of one dimensional elastic collision (co-efficient of restitution, $e = 1$) at macroscopic level when target is at rest $v = 0$ then depending upon masses of projectile and target following conclusions about the third law of motion can be drawn.

The magnitude of action and reaction

(a) The magnitude of reaction (final velocity of projectile after collision) is equal to action in magnitude in Eqs. (7, 10) only.

(b) The magnitude of reaction is not equal to action in Eqs. (8, 9, 11, 12).

(c) In Eq. (13) for definite action (u) there is no reaction as projectile comes to rest ($U = 0$). Thus under these conditions we conclude that

"To every action there may be a reaction but it may not always equal in magnitude to action."

The Direction of Action and Reaction :

(d) The direction of reaction is theoretically only opposite [*i.e.* projectile rebounds precisely in its original path] in Eqs. (7, 8, 9, 11) as indicated by negative sign.

(e) The reaction is not opposite in direction in Eqs. (10, 12)

(f) In Eq. (13) the question of direction does not arise as reaction itself is non-existent.

(g) If the projectile does not hit the target at center then it may be deflected sideways in a glancing collision, thus will not retrace its original path. The direction of reaction will not be opposite. Thus "The direction of reaction may not always be precisely opposite to that of action."

(h) Thus Newton's third law of motion is justified completely *i.e.* both in magnitude and direction in Eq. (7) only.

The third law of motion has been critically studied in elastic collisions ($e = 1$) in various cases individually for magnitude and direction. Hence it has been concluded that in such cases it must be generalised as

"To every action there may be reaction but not always equal and opposite to action."

If some experiments as described above are conducted specifically and quantitatively then, the validity of the generalised law will be established beyond any doubt.

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REFERENCES :

1. Resnick, R., Halliday, D., *Physics Part I* (Wiley Eastern Limited, New Delhi), pp. 87–90 and 215–222 forty second reprint (1992).
2. Sharma, A., to be published in *Acta Ciencia Indica*.
3. Sears, F.W., Zeemansky, M.W. and H.D. Young, *University Physics* (Narosa Publishing House, New Delhi) p.p. 148–154, Thirteenth reprint (1985).
4. Beiser, A., *Concepts of Modern Physics* (McGraw-Hill Book company, New York), pp. 483–84 fourth edition (1987).