The Origin of the Generalized Mass-Energy Equation $\Delta E = Ac^2\Delta M$ and Its Applications in General Physics and Cosmology

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Abstract

Einstein's (September 1905) derivation theorizes that when light energy (L) is emanated by a luminous body, its mass diminishes as $\Delta m = L/c^2$, and this equation is the speculative origin (without proof) of $\Delta E = c^2\Delta m$. The same derivation predicts that the mass of a luminous body inherently increases ($\Delta m = -0.034\,90L/cv + L/c^2$) when it emits light energy; in some cases, the mass of the body also remains the same ($\Delta m = 0$). An alternative equation, $\Delta E = Ac^2\Delta M$, has been suggested, that implies that energy emitted on the annihilation of mass (or vice versa) can be equal to, less than, or more than that predicted by $\Delta E = c^2\Delta m$. The total kinetic energy of fission fragments of U^{235} or Pu^{239} is found experimentally to be 20–60 MeV less than the Q value predicted by Δmc^2, which can be explained with $\Delta E = Ac^2\Delta M$ with a value of A less than one. $\Delta E = c^2\Delta m$ has not yet been confirmed in chemical reactions. The energy emitted by gamma-ray bursts (the most energetic event after the big bang) of duration 0.1–100 s is 10^{45} J, which cannot be explained by $\Delta E = \Delta mc^2$, similarly to the case of quasars. This can be explained with a high value of A, i.e., 2.57×10^{18}. The mass of the particle Ds (2317) discovered at SLAC is lower than current estimates, which can be explained with a value of A more than one. $\Delta E = Ac^2\Delta M$ explains that the mass of the universe of 10^{55} kg was created from a dwindling amount of energy (10^{-444} J or less), where A is 2.568×10^{-471} J or less, that in the end may reduce to small energy. It explains the big bang, the annihilation of antimatter in the hadron epoch, black holes, dark matter, etc. For the origin of inherent gravitational energy it implies that it is another form of mass like other energies, hence gravitation and mass are inseparable.

Key words: mass, energy, alternative equation $\Delta E = Ac^2\Delta M$, annihilation, Einstein's 27 September 1905 paper

1. EINSTEIN'S LIGHT-ENERGY–MASS EQUIVALENCE $\Delta M = L/c^2$ (SEPTEMBER 1905 PAPER)

The law of conservation of mass or energy has existed in the literature since the 18th century (or maybe even before that, informally). The French chemist Antoine Lavoisier (1743–1794) was the first to formulate such a law in chemical reactions. The very first idea of mass-energy interconversion, before Einstein's pioneering work,[2] that the kinetic energy of a cavity increases when it is filled with radiation in such a way that the *mass of the system appears to increase*, was given by Fritz Hasenohrl.[1] Then Einstein[2] calculated the relativistic form of kinetic energy [$KE_{rel} = (m_r - m_0)c^2$] in June 1905. From this equation at a later stage Einstein[3] derived the result $E_0 = m_0c^2$, where E_0 is the rest-mass energy, m_0 is the rest mass, and c is the velocity of light. Einstein also quoted the same method of derivation of rest-mass energy in his other works,[4] whereas in some other cases he completely ignored it.[5] Many other celebrated authors quote the derivation in the exact same simplified way.[6,7] Einstein[8] derived or speculated the relationship between mass annihilated (Δm) and energy created (ΔE), i.e., $\Delta E = \Delta mc^2$, in his paper widely known as September 1905. For the first time, the salient mathematical limitations and contradictions of this derivation have been pointed out and the alternative equation $\Delta E = Ac^2\Delta M$ has been proposed by the author. This method[8] is critically discussed below for understanding, and then its possible inconsistencies and the alternative equation ($\Delta E = Ac^2\Delta M$) are pointed out for the first time.

Einstein[8] perceived a luminous body at rest in coordinate system (x, y, z) whose energy relative to this system is E_0. System (ξ, η, ζ) is in uniform parallel translation with respect to system (x, y, z), and its origin moves along the x axis with velocity v. Let the energy of the body be H_0 relative to system (ξ, η, ζ). Let a system of plane light waves have energy ℓ relative to system (x, y, z), where the ray direction makes an angle φ with the x axis of the system. The quantity of light measured in system (ξ, η, ζ) has energy[2]

$$\ell^* = \frac{\ell\{1 - v/c\cos\varphi\}}{\sqrt{1 - v^2/c^2}} = \ell\beta\left\{1 - \frac{v}{c}\cos\varphi\right\}, \quad (1)$$

where

$$\beta = \frac{1}{\sqrt{1 - v^2/c^2}}.$$

Equation (1) was proposed by Einstein[2] in Section 8 as an analogous assumption without specific derivation or critical analysis.

Let this body emit plane waves of light of energy $0.5L$ (measured relative to (x, y, z)) in a direction forming an angle φ with the x axis. At the same time, an equal amount of light energy ($0.5L$) is emitted in the opposite direction ($\varphi + 180°$).

Consider the status of the body before and after the emission of light energy. Einstein says, "Meanwhile the body remains at rest with respect to system (x, y, z)." So the luminous body is not displaced from its position after the emission of light energy.

If E_1 and H_1 denote the energy of the body after the emission of light, measured relative to system (x, y, z) and system (ξ, η, ζ), respectively, using (1) we can write (equating initial and final energies in the two systems) that the energy of the body in system (x, y, z) is

$$E_0 = E_1 + 0.5L + 0.5L = E_1 + L. \quad (2)$$

In other words, the energy of the body with respect to system (x, y, z) before emission is equal to the energy of the body with respect to system (x, y, z) after emission plus the energy emitted (L):

$$H_0 = H_1 + 0.5\beta L\left[\left(1 - \frac{v}{c}\cos\varphi\right) + \left(1 + \frac{v}{c}\cos\varphi\right)\right]. \quad (3)$$

The energy of the body in system (ξ, η, ζ) is

$$H_0 = H_1 + \beta L. \quad (4)$$

In other words, the energy of the body with respect to system (ξ, η, ζ) before emission is equal to the energy of the body with respect to system (ξ, η, ζ) after emission plus the energy emitted (βL).

Subtracting (2) from (4), we have

$$(H_0 - E_0) - (H_1 - E_1) = L(\beta - 1). \quad (5)$$

In other words, the difference between the energy of the body in the moving system (ξ, η, ζ) and the energy of the body in system (x, y, z) before emission minus the difference between the energy of the body in the moving system (ξ, η, ζ) and the energy of the body in system (x, y, z) after emission is equal to $L[\beta - 1]$.

Einstein neither used nor mentioned in his calculation or description the relativistic variation of mass, which is given by

$$m_r = \beta m_0 = \frac{m_0}{\sqrt{1 - v^2/c^2}}, \quad (6)$$

where m_r is the relativistic mass and m_0 is the rest mass of the body, excluding the possibility that the velocity v is in the relativistic region. This equation existed before Einstein and was initially justified by Kaufmann[9] and more comprehensively by Bucherer.[10]

Further, Einstein[8] assumed the following relations (and tried to justify them at a later stage):

$$H_0 - E_0 = K_0 + C, \quad (7)$$

$$H_1 - E_1 = K_1 + C, \quad (8)$$

where K is the kinetic energy of the body and C is an additive constant that depends on the choice of the arbitrary additive constants in the energies H and E. Thus (5) becomes

$$K_0 - K = L\left[\frac{1}{\sqrt{1-v^2/c^2}} - 1\right]. \quad (9)$$

From here Einstein deduced, using the binomial theorem, without giving any justification, the relation between light energy (L) emitted and decrease in mass (Δm) as

$$\Delta m = \frac{L}{c^2} \quad (10)$$

or M_a (mass of body after emission) is equal to M_b (mass of body before emission) minus L/c^2.

Thus the mass of the body decreases when light energy is emitted. The body may also emit one, two, three, or many waves of different magnitudes of light energy ($0.499L$ and $0.501L$ or L, etc.) at different angles (different values of φ) and the velocity v may also be nonuniform. The derivation should be free of limitations and inconsistencies and should lead to the same result $\Delta m = L/c^2$ even under diverse conditions, as the law of conservation of mass and energy holds in all cases. These aspects are not addressed either by Einstein or by other scientists at all, and an attempt is made to do so here.

1.1 Detailed Discussion of Some Aspects of Einstein's Derivation

Einstein's derivation of $\Delta m = L/c^2$ and its generalization is in the form of a brief and compact discussion or note,[8] which contains no sections or subsections or numbered equations. Some significant deductions and conclusions are given by Einstein without explanation.

The law of conservation of mass-energy is a general law, having far-reaching importance and consequences, so a detailed and critical analysis of the derivation in all respects is absolutely necessary. Sharma[11–13] has initiated a logical discussion, which needs to be elaborated, taking all factors into account.

1.1.1 Einstein Used Classical Conditions of Velocity

If Einstein's remarks in his original paper[8] after the stage he derived (9) are quoted, then the above aspect (i.e., velocity is in the classical region) is crystal clear. We purposely quote Einstein's original text below in two parts in italics:

(i) "*The kinetic energy of the body diminishes as a result of the emission of light...*"

As the KE of the body is $m_0v^2/2$, this implies a decrease in the mass of the body because it is moving with constant velocity v.)

(ii) "*... and the amount of diminution is independent of the properties of the body.*"

The amount of diminution in KE is equal to (diminution in mass)$v^2/2$; Einstein regarded the velocity v as a constant in the derivation. Thus the magnitude of the diminution in mass depends on the original mass; if the mass does not change, the diminution in KE remains the same. The mass remains unchanged if the velocity is in the classical region. Thus if the velocity of the body is in the classical region, then the amount of variation of the mass of the body is independent of the velocity of the body (property of the body). But this is not so if the velocity v is in the relativistic region. In this case the mass increases as given by (6), which is neither taken into account nor mentioned at all. Hence Einstein has assumed that the velocity is in the classical region. So mass, diminution in mass, and hence kinetic energy are independent of the velocity of the body (property of the body), which is constant in the classical region. Further, to support this fact, Einstein has used the binomial theorem ($v << c$). He begins,[8] "Neglecting magnitudes of fourth and higher orders ..."

In (9) the magnitudes of the fourth and higher orders (v^4/c^4, v^6/c^6, ..., v^n/c^n) occur on application of the binomial theorem ($v << c$); these are neglected. Only magnitudes of second order (v^2/c^2) are retained in the calculations. In view of this, Einstein solved (9) as

$$K_0 - K = \frac{Lv^2}{2c^2}. \quad (11)$$

If Einstein had considered velocity in the relativistic region, then the equation

$$m_r = \beta m_0 = \frac{m_0}{\sqrt{1-v^2/c^2}} \quad (6)$$

would have been taken into account. Thus the mass of the body would have increased; but here only decrease in mass has been considered. In the mathematical and conceptual treatment Einstein did not mention this (increase in mass) at all. If the velocity of the body is not in the relativistic region, then it is in the classical region. The above remarks and exclusion of (6) specifically exclude this possibility and confirm that the velocity is in the classical region. We have

$$\begin{aligned} &KE \text{ of body before emission} \\ &\quad -KE \text{ of body after emission} \\ &= K_0 - K = L\left[\frac{1}{\sqrt{1-v^2/c^2}} - 1\right]. \end{aligned} \qquad (9)$$

Without giving any further mathematical explanation, Einstein[8] concluded, "If a body emits energy L in the form of radiation, its mass decreases by L/c^2."

Following is the obvious mathematical step not mentioned by Einstein in his brief note or paper,[8] as he has given the final result straightforwardly as in (10), i.e., $\Delta m = L/c^2$. Using the binomial theorem, (9) is written as (11), which can further be written as

$$\frac{M_b v^2}{2} - \frac{M_a v^2}{2} = \frac{Lv^2}{2c^2}. \qquad (11)$$

In other words, the mass of the body before emission (M_b) minus the mass of the body after emission (M_a) is equal to L/c^2, or

$$\Delta m = \frac{L}{c^2}. \qquad (10)$$

This means M_a (mass of body after emission) is equal to M_b (mass of body before emission) minus L/c^2.

Here L is the energy emitted by the luminous body, i.e., the difference between the magnitudes of final energy (L_{final}) and initial energy ($L_{initial}$); thus (10) in a more transparent way can be written as

$$M_b - M_a = \frac{L_{final} - L_{initial}}{c^2}. \qquad (10)$$

If $L_{initial}$ is regarded as zero, then light energy emitted (ΔL) equals L_{final}, L (say).

Thus this implies that when a body emits light energy, then its mass decreases; i.e., the mass is annihilated into light energy. Thus the law of conservation of light energy emitted and mass annihilated is established. Then Einstein speculated that the conservation law exists not only between light energy emitted and mass annihilated but also between all energies and masses, without scientific justification or logical embellishment, as a postulate. This aspect is discussed separately.

1.1.2 Equation (10) Can Also Be Obtained if a Single Wave of Light Energy L Is Emitted

This case has been discussed by neither Einstein nor others. Consider that a body is placed in the system (x, y, z) and emits a single wave of light energy L, which is perpendicular to the ray direction (φ = 90°). It is observed in system (ξ, η, ζ) moving with uniform relative velocity v, exactly in the same way as in Einstein's derivation. In that case, (3) can be written as

$$\begin{aligned} H_0 &= H_1 + \beta L\left[1 - \frac{v}{c}\cos 90°\right] \\ &= H_1 + \beta L. \end{aligned} \qquad (4)$$

Also, we have

$$E_0 = E_1 + L. \qquad (2)$$

Now, proceeding as in (5)–(10), we get

$$\Delta m = \frac{L}{c^2}. \qquad (10)$$

In other words, M_a (mass of body after emission) equals $-L/c^2$ plus M_b (mass of body before emission), which is the same result as obtained by Einstein.

1.1.3 Einstein's Condition: The Body Remains at Rest Before and After Emission of Light Energy or the Momentum of the Body Before Emission Is Equal to the Momentum After Emission

Einstein did not even mention the term *conservation of momentum* in his derivation at all. However, according to Einstein's original remarks (after describing the emission of light energy by a luminous body), "Meanwhile the body remains at rest with respect to system (x, y, z)."

Einstein's condition is satisfied in numerous cases when a luminous body emits energy and remains at rest. The body remains at rest before and after emitting light energy L since recoil is vanishingly small or zero, so the initial momentum of the luminous body is equal to the final momentum. The extent of recoil also depends on the resistive forces (frictional, gravitational, atmospheric, etc.) present in the system. The body may remain at rest after emitting one, two, or more waves simultaneously with energy different from 0.5L. In such cases, the initial and final momenta of the luminous body are equal since the net change after emitting the energy is negligibly small.

Thus the body remains at rest, as required by Einstein's constraint. Thus Einstein has considered conservation of momentum of the system (body and waves), not conservation of only waves (they must be the same and in opposite directions). In the paper Einstein did not even use the term *conservation of momentum*; this justifies the fact that if the body remains at rest before and after emission, then momentum is conserved.

Further, in the description of a light-emitting body, its mass is regarded as in the classical region. For example, Einstein perceived that the body (not the particle or the wave) remains at rest before and after emitting energy. If a wave or a particle emits light energy, then it will not remain at rest, as per Einstein's condition. Thus the mass may be of the order of 10 g or less (such that the body remains at rest after emitting light energy, as mentioned by Einstein) or more (the body may be heavier than 100 kg unreservedly). The law of conservation of energy holds for all bodies, particles or waves; thus the derivation must be general and applicable to all cases.

Conservation of momentum, which is not at all mentioned by Einstein, holds when a body emits light energy. For example, if a light wave is emitted in the visible region ($f = 4 \times 10^{14}$ Hz) with energy (hf) 2.6×10^{-19} J, it is unable to cause any observable recoil (however, it tends to do so) in a body of mass in the classical region as used or intended by Einstein. As already mentioned, the recoil also depends on resistive forces present in the system. The law of conservation of momentum is obeyed in this case, as in the previous case considered by Einstein[8] to derive (10). The body remains at rest before and after emitting energy, which is Einstein's main condition in the above derivation.

Analogously, consider a person firing a bullet from a gun. As the bullet moves forward, the person recoils observably, and the law of conservation of momentum is obeyed (the forward momentum of the bullet equals the backward momentum of the person). However, the exact velocity of recoil also depends on resistive forces (frictional, atmospheric, and gravitational) present in the system. Then the person fires a shot from a toy gun, and this shot is unable to cause any observable recoil. (However, the system tends to recoil.) The initial momentum remains equal to the final momentum. The law of conservation of momentum (in one wave or two waves or more) holds equally in these cases; the energy of the waves is unable to recoil the body observably. In this case, the initial momentum and the final momentum (after emitting light waves) of the luminous body are also equal, as the change in momentum is vanishingly small.

The nondisplacement of the body from its original position during and after the emission of light energy may be regarded as the simplest or special case. However, the law of conservation of mass-energy (which is the ultimate result) holds in all cases when the velocity of the body may be zero, uniform, or nonuniform (the motion may be accelerated or nonaccelerated) with respect to system (x, y, z) and system (ξ, η, ζ). As already mentioned, the body also *remains at rest* if the light energy emitted by the luminous body in one, two, or more waves at different angles has different magnitudes of energy. Thus Einstein has discussed the simplest case.

When two waves are emitted, the body remains at rest: the momentum of the wave emitted at 0° plus the momentum of the body is equal to the momentum of the wave emitted at 180° plus the momentum of the body.

Einstein has assumed that the body remains at rest before and after the emission of light energy. Thus the momentum of the wave emitted at 0° is equal to the momentum of the wave emitted at 180°. Hence we have the result that the body remains at rest (conditions imposed by Einstein in the paper on derivation).

When one wave is emitted, the body remains at rest. Consider the initial momentum: the initial momentum of the body plus the momentum of the wave is equal to zero (as the body is at rest) plus zero (as no wave is emitted).

Then consider the final momentum: the final momentum of the body plus the momentum of the wave (emitted at an angle). Then we have momentum conservation (initial = final): zero is equal to the final momentum of the body plus the momentum of the wave (emitted at an angle), or the momentum of the wave (emitted at an angle) is equal to the negative of the momentum of the body.

But even if the magnitude of recoil of the body is too small to be measured, the body does possess momentum. Momentum is conserved in this case also.

Similarly, conservation of two or more waves can be demonstrated. In all cases momentum is conserved, but Einstein did not mention conservation of momentum in his derivation; instead he perceived that the body remains at rest after the emission of waves.

Consider the case when four waves at angles φ, φ + 180°, θ, and θ + 180° have magnitude of light energy $0.255L$ and $0.245L$. The angles at which the four waves are emitted are 0°, 190°, 75°, and 250°.

Consider the initial momentum: the momentum of the body plus the momentum of the waves is equal to zero (as body is at rest) plus zero (as no waves are emitted).

Then consider the final momentum: the momentum of the body plus the sum of the momenta of the four waves.

Then we have conservation of momentum: zero is equal to the momentum of the body plus the sum of the momenta of the four waves, or the momentum of the body is equal to the negative of the sum of the momenta of the four waves.

Thus the momentum of the body is the net effect of the momenta of the waves.

The law of conservation of momentum (and energy) also holds if the body moves.

1.2 Einstein Did not Compare the Relativistic Kinetic Energy of a Slowly Accelerated Electron, $K_f - K_i = W = m_0c^2\{1/(1 - v^2/c^2)^{1/2} - 1\} = m_rc^2 - m_0c^2$, and the Kinetic Energy from the Light-Energy–Mass Equation, $K_0 - K = L\{1/(1 - v^2/c^2)^{1/2} - 1\}$

According to the work-energy theorem, work done is equal to change in kinetic energy; i.e., work done (W) is equal to final kinetic energy (K_f) minus initial kinetic energy (K_i).

If the body is moving with uniform velocity ($a = 0$), then the initial KE is equal to the final KE, so W is zero; also in this case acceleration is zero, so force ($F = ma$) and work done ($W = FS$) are also zero. Then Einstein introduced the special theory of relativity[2] as

$$K_f - K_i = W = m_0c^2\left[\frac{1}{\sqrt{1-V^2/c^2}} - 1\right] \qquad (12)$$
$$= m_rc^2 - m_0c^2,$$

where V is the variable velocity (slowly accelerated motion) and the other terms have been defined earlier.

Further, in (9), L is ΔL, i.e., the difference in initial and final energies.

Einstein remarks (after indirectly justifying that the velocity v is in the classical region), "Moreover, the difference $K_0 - K$, like the kinetic energy of the electron, depends upon the velocity."

These may be regarded as just passing remarks, as the right-hand sides of both equations depend on velocity (v, constant and V, variable). In his original thesis[8] Einstein correctly did not equate or mathematically compare the equations, i.e., (9) and (12), due to the following reasons.

Equations from different derivations cannot be compared or equated unless they have the same mathematical and conceptual nature or basis and they discuss identical scientific aspects. Equations from different derivations cannot be mathematically equated just because they have the same dimensions. Energy has different forms, e.g., heat energy, light energy, sound energy, electrical energy, kinetic energy, etc., but has the same dimensions. The dimensions of torque and work or energy are the same, i.e., ML^2T^{-2}, but they cannot be equated. Mathematically, torque = $rF\sin\theta$, a vector quantity, and work = $FS\sin\theta$, a scalar quantity; thus in one case θ is the angle between F and r (position vector) and in the second case θ is the angle between F and S (displacement). Hence, when the angle θ is 0°, then the torque is zero but the work done is maximum; when $\theta = 90°$, the torque is maximum and the work done is zero. Thus even if two physical quantities have the same dimensions, they may represent two different aspects, and hence cannot be equated.

There are different ways to measure the change in kinetic energy in (9) and (12). In (9), the change in kinetic energy is equal to the initial kinetic energy (before the emission of light energy) minus the final kinetic energy (after the emission of light energy). In (12), according to the work-energy theorem, the change in kinetic energy is equal to the final kinetic energy (after the increase in mass) minus the initial kinetic energy (original mass). Thus the expressions for change in KE in both the equations, i.e., (9) and (12), are different.

Consider the decrease or increase in mass. In (9), due to the emission of light energy, the final mass of the body *decreases*, where the decrease in mass is converted into energy. Thus the final mass of the body is less than the original mass. In (12), when an external force acts on the body and the velocity is in the relativistic region, the mass of the body *increases* and is known as the relativistic mass:

$$m_r = \frac{m_0}{\sqrt{1-V^2/c^2}}. \qquad (6)$$

The energy that is externally supplied to accelerate the body is converted into mass. Thus in (12) the mass of the body *increases* and in (9) the mass of the body *decreases*.

Consider constant and variable velocity. In (9) the velocity v with which system (ξ, η, ζ) moves with respect to system (x, y, z) is constant (thus motion is nonaccelerated). But in (12), which as quoted by Einstein[2] himself represents the motion of a slowly accelerated electron, the velocity V is variable. Hence in (12) the velocity is variable, i.e., the motion is *accelerated*, and in (9) the velocity is constant, i.e., the motion is *unaccelerated*.

Consider the velocity in the classical region and the relativistic region. In (9) and subsequent equations, Einstein has done his mathematical treatment under classical conditions. For example, Einstein has mentioned that the *KE* of the body decreases (hence mass decreases, as velocity v is constant) and the relativistic increase in mass with velocity is not taken into account at all, so the velocity is in the classical region. Had the velocity been in the relativistic region, the increase in mass would have been taken into account as in (6). Thus the velocity is in the classical region. In (12), the relativistic *KE* of the slowly accelerated electron has been calculated using (6):

$$m_r = \frac{m_0}{\sqrt{1-V^2/c^2}}, \tag{6}$$

which clearly implies that the velocity V is in the relativistic region. In the case $v = 0$ or v is negligible compared to c, $m_r = m_0$. If the velocity is in the classical region, then (12) is simply $m_0V^2/2$, which implies no change in mass. Hence in (12) the velocity V is in the relativistic region ($V \sim c$), whereas in (9) the velocity v is in the classical region ($v \ll c$)

In (9) energy is emitted to the surroundings and in (12) energy is taken from the surroundings. In (9) and subsequent equations it is implied that the mass of the body *decreases* and energy is emitted to the surroundings as light energy. In (12), when the mass is slowly accelerated to the relativistic region (V becomes comparable to c), then the mass of the body *increases*. The increase in mass is due to energy taken from the surroundings. Hence in (12) energy is *absorbed* by the body from the surroundings, and in (9) energy is *emitted* by the body to the surroundings. Thus (9) and (12) are based on entirely diverse and conflicting concepts and hence cannot be mathematically compared simply because the dimensions are the same.

1.2.1 Justification of Transformation from (5), (7), and (8) to (9)

Einstein[8] correctly did not draw any direct conclusion from (9) and (12) simultaneously or equate these two equations. Einstein tried to justify two aspects simply on the basis of the fact that the right-hand sides of (9) and (12) depend on velocity (the nature of the equations is entirely different). First, the transformation of (5), (7), and (8) to (9) is logical, which means the justification of the introduction of the arbitrary additive constant C. Second, he assumed that the arbitrary additive constant C does not change during the emission of light.

1.3 Infinitesimally Small Variations in Parameters (φ and L) Cause Drastic Changes in Characteristics and Concepts; These Are not Discussed by Einstein

1.3.1 One Wave Is Emitted

Consider the case when the angle formed by a single wave is 89° or 91° not 90°.

When φ = 89°, consider a body being placed in system (x, y, z) and emitting a single wave of light energy L, making an angle 89° with the direction of propagation, and observed in system (ξ, η, ζ), which is moving with uniform relative velocity v. Then

$$\begin{aligned} H_0 &= H_1 + \beta L\left(1 - \frac{v}{c}\cos 89°\right) \\ &= H_1 + \beta L\left(1 - 0.017\,452\,406\frac{v}{c}\right). \end{aligned}$$

Now, proceeding as in (5)–(10), we get

$$\Delta m = -0.034\,90\frac{L}{cv} + \frac{L}{c^2}. \tag{13}$$

In other words, M_a (mass of the body after emission) is equal to $0.034\,90L/cv$ minus L/c^2 plus M_b (mass of the body before emission), which implies that the mass of the body increases when light energy is emitted. On the right-hand side, the term $(-0.034\,90L/cv + L/c^2)$ is always negative, as the velocity v is in the classical region. The percentage difference between (10) and (13) is $3.49c/v$ or 1.047×10^8 if v is 10 m/s (in the classical region).

Similarly, if the angle made is 91° (cos 91° = −0.017 452 4) with the direction of propagation, then

$$\begin{aligned} \Delta m &= +0.034\,90\frac{L}{cv} + \frac{L}{c^2}, \\ M_a &= -0.034\,90\frac{L}{cv} - \frac{L}{c^2} + M_b, \end{aligned} \tag{14}$$

which implies that the mass of the body decreases when light energy is emitted. But this decrease in mass is much more than (10). The percentage difference between (10) and (14) is $3.49c/v$ or 1.047×10^8 if v is 10 m/s (in the classical region).

If the angles considered in the derivation are 90°, 89°, and 91°, then the results are altogether different. This implies that the angle at which light energy is emitted is a significant factor in Einstein's derivation. The law of conservation of mass-energy should not depend on angles like this.

1.3.2 Two Waves Are Emitted

When the magnitude of energy emitted is slightly different from $0.5L$, it is assumed that the body emits plane waves of light of energy $0.4999L$ along the x axis, i.e., $\varphi = 0°$. The other wave of light energy $0.5001L$ is emitted in exactly the opposite direction, i.e., forming an angle 180° (Einstein has assumed light energy $0.5L$ in both cases). Then the equation equivalent to (3) becomes

$$H_0 = H_1 + 0.501\beta L\left(1-\frac{v}{c}\cos 0°\right) + 0.499\beta L\left(1-\frac{v}{c}\cos 180°\right).$$

After proceeding in a similar way as above, we get

$$K_0 - K = -0.002\frac{v}{c} + \frac{Lv^2}{2c^2}, \tag{15}$$

or

$$\begin{aligned}\Delta m &= \text{Mass of body before emission } (M_b) \\ &\quad - \text{Mass of body after emission } (M_a) \\ &= -0.004\frac{L}{cv} + \frac{L}{c^2},\end{aligned} \tag{16}$$

or

$$M_a = 0.004\frac{L}{cv} - \frac{L}{c^2} + M_b,$$

which implies that when light energy L ($0.499L$ along the x axis, $0.501L$ in the opposite direction) is emitted, then the mass of the luminous body increases ($v \ll c$), so $0.004L/cv$ is always more than L/c^2. This is contrary to Einstein's deduction, i.e., (10). The percentage difference between (10) and (16) is $0.4c/v$ or 1.2×10^7 if the velocity is 10 m/s, i.e., in the classical region.

When the magnitude of the angle at which one wave is emitted is 181° rather than 180° (as considered by Einstein) and the other is at 0°, then

$$\begin{aligned}H_0 &= H_1 + 0.5\beta L\left(1-\frac{v}{c}\cos 0°\right) \\ &\quad + 0.5\beta L\left(1-\frac{v}{c}\cos 181°\right) \\ &= H_1 + \beta L\left(1-0.000\,076\,15\frac{v}{c}\right),\end{aligned}$$

$$K_0 - K = -0.000\,076\,15\frac{Lv}{c} + \frac{Lv^2}{c^2}, \tag{17}$$

$$\Delta m = -0.000\,152\,3\frac{L}{cv} + \frac{L}{c^2}, \tag{18}$$

or

$$M_a = -0.000\,152\,3\frac{L}{cv} - \frac{L}{c^2} + M_b,$$

which implies that the mass of the body increases when light energy is emitted, as in the classical region (10 m/s) it is more than L/c^2. If the angle $\varphi = 181°$, this difference of 1° in the angle causes drastic changes in the results. Earlier, Einstein considered that if the angle φ is 180° then the mass of the body decreases on emission. If the angles considered in the derivation are 90°, 89°, and 91°, then the results are altogether different. This implies that the angle is a significant factor in Einstein's derivation. The law of conservation of mass-energy should not depend on angles like this.

2. SOME FEASIBLE CASES NEGLECTED BY EINSTEIN AND CONTRADICTIONS OF THE DERIVATION

The only condition put by Einstein on the luminous body is that it must *remain at rest* before and after the emission of light energy. This condition is satisfied in numerous cases and hence Einstein's derivation is applicable. Einstein has discussed the simplest or typically special case when two light waves (of equal energy $0.5L$) are emitted in opposite directions, which is just one of numerous possible cases. But the following genuine possibilities (which are absolutely essential) have been discussed neither by Einstein nor by others.

Many Waves or a Single Wave

The sources of light may emit many light waves simultaneously or just a single wave; then the angle φ and the magnitude of light energy (L) may be different. Einstein has discussed the simplest and special case when two waves are emitted. Realistically, it is a particular case and the body may emit a single or many waves depending on its characteristics and *remain at rest*. Bodies can be fabricated to have a wide range of parameters to check all aspects of the derivation.

Angle at Which Light Waves Are Emitted

Einstein considered just two angles, φ and $\varphi + 180°$, at which light waves are emitted. The body remains at rest if it emits light energy at numerous angles. This is Einstein's only condition regarding this; i.e., the body remains at rest before and after emission. In general, the luminous body can emit light waves at various angles. Even slight variations in angle(s) cause very significant changes in concepts, as discussed in (10), (14), (16), and (18). But this aspect has not been taken into account by Einstein.

Magnitude of Light Waves

In the derivation Einstein has considered that the total light energy emitted by the luminous body is L, which is emitted in two waves, each having energy $0.5L$, moving in opposite directions. It can be $0.499L$ and $0.501L$ or something else. Further, the luminous body can emit a large number of waves; then the light energy L will be distributed among the waves equally or unequally and the body *remains at rest*. If the energy emitted is different from $0.5L$, then the results are different from $\Delta m = L/c^2$, as in (21), (24), and (26).

Velocity v May Be Nonuniform

Einstein considered systems in uniform relative motion v with respect to each other. The system may have uniform or nonuniform velocity, may be in the classical or the relativistic region, or may be at rest, but the law of conservation of mass and energy holds under all circumstances. Thus the derivation should have been applicable for all values of the velocity.

Application of the Binomial Theorem

Further, this derivation is stage sensitive, as $\Delta m = L/c^2$ is only obtained if the binomial theorem is applied at a particular stage, i.e., (9). The same is also applicable to (1) at a much earlier stage; if it is applied here, then we get contradictory results; i.e., $\Delta m\ (M_b - M_a) = 0$ instead of $\Delta m = L/c^2$, as shown in (29). Thus in this derivation the application of the binomial theorem makes or mars the law of conservation of energy, so the derivation is inconsistent. However, in the derivation of the relativistic form of kinetic energy [$KE_{rel} = (m_r - m_0)c^2$], the binomial theorem may be applied at any stage, but the result is the same, i.e., the classical form of kinetic energy ($m_0v^2/2$).

Nature and Limitations of (1)

The central equation in the derivation is (1), which gives the variation of light energy with velocity. This equation has been quoted (but without mathematical details) by Einstein in a paper in which he gave the special theory of relativity (Ref. 2, Sect. 8). The nature of this equation is different from the equations that existed before Einstein's work, e.g., relativistic variation of mass [$m_r = m_0/(1 - v^2/c^2)^{1/2}$], length contraction [$L = L_0(1 - v^2/c^2)^{1/2}$] and time dilation [$T = T_0(1 - v^2/c^2)^{1/2}$)]. This equation also has serious mathematical limitations. All the above aspects are described below along with their impacts and consequences.

2.1 Violation of Law of Conservation of Mass and Energy, i.e., Mass and Energy Are Created out of Nothing Simultaneously

This case can be discussed considering the cases when the luminous body emits one or more (even or odd) waves.

Four Waves Are Emitted

Consider the case of four waves at angles φ, $\varphi + 180°$, θ, and $\theta + 180°$, with magnitude of light energy $0.255L$ and $0.245L$. The angles at which the four waves are emitted are 0°, 180°, 75°, and 255° (75° + 180°). In view of this (3) can be written as

$$\begin{aligned} H_0 = H_1 &+ 0.255L\beta\left(1-\frac{v}{c}\cos 0°\right) \\ &+ 0.245L\beta\left(1-\frac{v}{c}\cos 180°\right) \\ &+ 0.255L\beta\left(1-\frac{v}{c}\cos 75°\right) \\ &+ 0.245L\beta\left(1-\frac{v}{c}\cos 255°\right) \\ = H_1 &+ L\beta\left(1-0.012\,588\frac{v}{c}\right). \end{aligned} \tag{19}$$

Using (2) and (19), we get, as in previous cases,

$$K_0 - K = L\left(-0.012\,588\frac{v}{c}+\frac{v^2}{2c^2}\right), \tag{20}$$

$$\Delta m\,(M_b - M_a) = -0.025\,176\frac{L}{cv}+\frac{L}{c^2}, \tag{21}$$

or

$$M_a\text{(mass after emission)} = 0.025\,176\frac{L}{cv} - \frac{L}{c^2} + M_b\text{(mass before emission)}. \tag{22}$$

The above equation is derived by applying the binomial theorem ($v << c$); under this condition Δm from (21) is always negative. The percentage difference between (10) and (21) is 3.7764×10^7 (if v is 10 m/s). Hence, from (22), the following two conclusions are absolutely clear:

(a) The body is emitting light energy, L (say, a very significant amount of energy) continuously.
(b) Simultaneously as light energy L is emitted, the mass of the body (matter) is also increasing.

From this interpretation the following two conclusions are clear:

(i) The mass of the body increases when the body emits light energy L. Einstein's 1905 derivation also predicts that the luminous body emits energy and simultaneously the mass of the body increases. The creation of both energy and mass is at a cost of nothing or cipher. This is a clear contradiction of the law of conservation of mass and energy.
(ii) We have a contradiction with the relativistic variation of mass. According to the relativistic variation of mass, i.e., (6), or $m_r = \beta m_0$, mass increases when the velocity of the body is comparable with the speed of light; this aspect was also used by Einstein, from existing literature, in the paper in which he introduced the theory of relativity.[2] This result is contradicted by Einstein himself. This derivation implies that the mass of the body can also increase if it moves with classical velocity ($v << c$) and surprisingly emits light energy (which Einstein established is a form of mass). Thus this derivation also contradicts the relativistic variation of mass with velocity.

Five Waves Are Emitted

If five waves are emitted such that the waves make angles 0°, 90°, 145°, 300°, and 340° with the x axis and each wave carries energy equal to $0.2L$, then an equation equivalent to (3) can be written as

$$\begin{aligned} H_0 = H_1 &+ 0.2L\beta\left(1-\frac{v}{c}\cos 0°\right) \\ &+ 0.2L\beta\left(1-\frac{v}{c}\cos 90°\right) \\ &+ 0.2L\beta\left(1-\frac{v}{c}\cos 145°\right) \\ &+ 0.2L\beta\left(1-\frac{v}{c}\cos 300°\right) \\ &+ 0.2L\beta\left(1-\frac{v}{c}\cos 340°\right) \\ = H_1 &+ 0.2L\beta\left(5-0.620\,543\frac{v}{c}\right). \end{aligned} \tag{23}$$

Using (2) and (23), we get, as in previous cases,

$$\begin{aligned} K_0 - K &= L\left[\beta\left(1-0.124\,108\,1\frac{v}{c}\right)-1\right], \\ \Delta m &= -0.2482\frac{L}{cv}+\frac{L}{c^2}, \\ M_a &= 0.2482\frac{L}{cv}-\frac{L}{c^2}+M_b. \end{aligned} \tag{24}$$

That is, the mass of the body increases, which is contradictory, as discussed earlier in the case of (13) and others.

Six Waves Are Emitted

Consider the case when six waves are emitted by the luminous body, making angles 0°, 180°, 75°, 255°, 60°, and 69°; the magnitudes of light energy emitted along these directions are $0.25L$, $0.24L$, $0.25L$, $0.24L$, $0.01L$, and $0.01L$, respectively. In view of this, (3) can be written as

$$\begin{aligned} H_0 = H_1 &+ 0.25L\beta\left(1-\frac{v}{c}\cos 0°\right) \\ &+ 0.24L\beta\left(1-\frac{v}{c}\cos 180°\right) \\ &+ 0.25L\beta\left(1-\frac{v}{c}\cos 75°\right) \\ &+ 0.24L\beta\left(1-\frac{v}{c}\cos 225°\right) \\ &+ 0.01L\beta\left(1-\frac{v}{c}\cos 60°\right) \\ &+ 0.01L\beta\left(1-\frac{v}{c}\cos 69°\right). \end{aligned} \tag{25}$$

Substituting various values and rearranging terms, we have

$$H_0 = H_1 + L\beta\left(1 - 0.021\,17\frac{v}{c}\right).$$

In terms of change in mass as calculated in (24) and other equations,

$$\Delta m = -0.042\,34\frac{L}{cv} + \frac{L}{c^2}, \qquad M_a = 0.042\,34\frac{L}{cv} - \frac{L}{c^2} + M_b. \tag{26}$$

Hence we have a similar contradictory result as discussed in the case of (24) or (13) and others.

Likewise, results can be discussed if the luminous body emits many waves.

2.2 Violation of Law of Conservation of Mass-Energy; i.e., Energy Is Created out of Nothing in Einstein's Derivation

As already mentioned, v is the velocity with which the coordinate system (ξ, η, ζ) moves with respect to system (x, y, z). If the velocity v is regarded as zero ($v = 0$), i.e., system (ξ, η, ζ) is at rest and the body emits light energy L as before, then (1) becomes

$$\ell^* = \ell. \tag{27}$$

Rewriting (3) in view of the above, we get

$$H_0 = H_1 + 0.5L + 0.5L = H_1 + L. \tag{28}$$

From (28) and (2),

$$(H_0 - E_0) - (H_1 - E_1) = 0.$$

Now, proceeding in a similar way as earlier, we get

$$K_0 - K = 0,$$
$$\Delta m\,(M_b - M_a) = 0,$$

or

$$\text{Mass of body before emission } (M_b) = \text{Mass of body after emission } (M_a). \tag{29}$$

Thus, when system (ξ, η, ζ) is at rest and the luminous body emits energy (say, a significant amount), then its mass remains constant, i.e., energy is being emitted out of nothing; the law of conservation of mass and energy is clearly contradicted in Einstein's derivation. In all nuclear reactions, chemical reactions, etc., mass is converted into energy and energy is obtained at the cost of mass. But here the derivation predicts that the energy L (say a significant amount of energy) is created without loss of mass at all, as in (29) the mass remains the same, which is inconsistent in all respects.

Just for complete scrutiny, if the same condition, i.e., $v = 0$, is applied to (9), then

$$0 = 0.$$

In this case, there is no nonzero term to interpret how light energy L is emitted. Thus the body is emitting light energy light L that is created out of nothing or cipher, as the mass of the body remains the same.

2.3 Contradiction of $\Delta m = L/c^2$ Itself, i.e., Energy Emitted Is More than $L = \Delta mc^2$

Similarly, consider that the source emits light energy L in a single wave such that it makes an angle 180° with the x axis:

$$H_0 = H_1 + \beta L\left(1 + \frac{v}{c}\right). \tag{30}$$

Now, proceeding in a similar way as earlier, we get

$$K_0 - K = \beta L\left(1 + \frac{v}{c}\right) - L,$$

or

$$\Delta m = \frac{2L}{cv} + \frac{L}{c^2}. \tag{31}$$

If the value of v is 10 m/s, then (31) becomes

$$\Delta m\,(M_b - M_a) = \frac{2L}{3\times10^9} + \frac{L}{9\times10^{16}}, \qquad M_a = -\frac{2L}{3\times10^9} - \frac{L}{9\times10^{16}} + M_b. \tag{32}$$

Also, in this case, (10) becomes

$$\Delta m\,(M_b - M_a) = \frac{L}{9\times10^{16}}, \tag{32}$$

or

$$M_a = -\frac{L}{9\times10^{16}} + M_b. \tag{33}$$

The magnitude of (32) is much greater than that of (10). Now the percentage difference between (32) and Einstein's original equation, i.e., (10), is $200c/v$ or 6×10^9 (if v = 10 m/s). Thus the results given by this derivation are self-contradictory. Many such examples can be quoted in this regard.

3. IF THE BINOMIAL THEOREM IS APPLIED TO (9), THE LAW OF CONSERVATION OF MATTER AND ENERGY IS ORIGINATED, AND IF THE BINOMIAL THEOREM IS APPLIED TO (1), THEN THE LAW IS CONTRADICTED

If the binomial theorem is applied to (9), as in the case of Einstein's derivation, then, as already mentioned, the result is

$$\Delta m = \frac{L}{c^2}. \tag{10}$$

When the binomial theorem is applied at (1), i.e., $v \ll c$, then

$$\ell^* = \beta\ell\left(1 - \frac{v}{c}\cos\varphi\right) = \ell. \tag{27}$$

Thus the subsequent equations can be written as

$$E_0 = E_1 + L, \tag{2}$$

$$H_0 = H_1 + L, \tag{4}$$

$$(H_0 - E_0) - (H_1 - E_1) = 0,$$

$$K_0 - K = 0 \text{ or } \Delta m = 0,$$

$$\text{Mass of body before emission } (M_b) = \text{Mass of body after emission } (M_a). \tag{29}$$

Equation (29) implies that if the body emits light energy, then its mass remains the same; this is contrary to (10), given by Einstein, which implies that when light energy is emitted, the mass of the body decreases. Thus this derivation has a limitation, because the stage at which the binomial theorem is applied makes or breaks the law of conservation of mass and energy. However, the law of conservation of mass-energy holds good under all conditions and must not depend upon the stage at which the binomial theorem (a mathematical tool) is applied. The results of this derivation are affected by other factors as well.

3.1 Velocity Constraints on Einstein's Light-Energy–Mass Equivalence ($\Delta m = \Delta L/c^2$)

From the above discussion it is evident that Einstein's derivation has an overdependence on velocity. It only holds if the velocity is in the classical region, gives contradictory results if the body is at rest (v = 0), and is not applicable if v is in the relativistic region or $v \to c$.

When the velocity is in the classical region ($v \ll c$), the binomial theorem is applicable to (9):

$$K_0 - K = L\left[\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right], \tag{9}$$

or

$$\Delta m = \frac{L}{c^2}, \tag{10}$$

or

$$\text{Mass of body after emission } (M_a) = \text{Mass of body before emission } (M_b) - \frac{L}{c^2}.$$

Thus the mass of the body decreases when light energy is emitted.

When the body is at rest, $v = 0$, or

$$K_0 - K = 0 \text{ or } \Delta m\,(M_b - M_a) = 0,$$

or

$$\text{Mass of body before emission } (M_b) = \text{Mass of body after emission } (M_a). \tag{29}$$

Thus when system (ξ, η, ζ) is at rest and the luminous body emits energy (say, a significant amount), its mass remains constant; i.e., energy is being emitted out of nothing. The law of conservation of mass and energy is clearly contradicted in Einstein's derivation of $\Delta m = \Delta L/c^2$.

When the velocity is in the relativistic region or $v = c$, there is relativistic variation of the mass of the body and (6) is applicable. This factor is not at all taken into account in the mathematical and conceptual treatment. Hence Einstein's light-energy–mass equation is not applicable in this case.

Thus there are constraints due to the velocity, angle, and magnitude of light energy on the derivation. In fact, the law of conservation of mass and energy is basic and should not have constraints like this.

3.2 Comparison of Einstein's Light-Energy–Mass Equation and Relativistic Form of *KE*

Using the relativistic variation of mass,

$$m_r = \beta m_0 = \frac{m_0}{\sqrt{1 - v^2/c^2}}, \qquad (6)$$

Einstein obtained the relativistic form of *KE*:

$$dK = Fdx \text{ or } (KE)_{rel} = (m_r - m_0)c^2. \qquad (12)$$

If the binomial theorem is applied to (12), then

$$(KE)_{rel} \text{ when } v << c = (KE)_{classical} = \frac{m_0 v^2}{2}, \qquad (34)$$

which is the usual form of *KE* as obtained by other methods. In the equation $KE = m_0 v^2/2$, m_0 is the rest mass. It cannot be interpreted that if m_0 decreases then *KE* increases.

If the binomial theorem is applied to (6), then

$$m = m_0,$$

or

$$dK = Fdx = \frac{dp}{dt}dx \text{ or } K = \frac{m_0 v^2}{2}. \qquad (34)$$

Thus if we apply the binomial theorem to (6) or (12), we get the same equation for kinetic energy, $K = m_0 v^2/2$, i.e., (34).

Analogously, if the binomial theorem is applied to (1) and (9), then different values of the change in mass, i.e., $\Delta m = 0$ or $\Delta m = L/c^2$, are obtained (in one case the mass remains the same and in the other the mass decreases). In fact, the result should have been the same as in (34), regardless of whether the binomial theorem is applied to (1) or (9). This clearly highlights the limitations of Einstein's derivation, which have gone unnoticed by the scientific community.

4. EQUATION (10) IS BASED ON AN INCONSISTENT EQUATION

By (1), we have inconsistency of dimensional homogeneity when $v \to c$. The central equation in this derivation is (1), which was quoted (without mathematical details) by Einstein himself in Section 8 of the paper in which he introduced the special theory of relativity.[2]

If $\varphi = 0°$, then (1) becomes

$$\ell^* = \ell \frac{1 - v/c}{\sqrt{1 - v^2/c^2}}. \qquad (35)$$

This equation is applicable when the velocity v is uniform; if the velocity is nonuniform in the interval, its application requires estimations of subintervals when the velocity is uniform. If system (ξ, η, ζ) moves with velocity equal to that of light, i.e., $v = c$, which realistically means that the velocity v tends to c, i.e., $v \to c$ (some quasars or other heavenly bodies may attain such high velocities), then

$$\ell^* = \frac{0}{0}.$$

which is undefined, or ℓ^* tends to 0/0, which has the same meaning.

The dimensions of the left-hand side are ML^2T^{-2} [energy], and the dimensions of the right-hand side are undefined. It is an inherent requirement that an equation must obey the principle of dimensional homogeneity,[14,15] but this is not so in the case of (1) under this particular condition. Hence, under this condition, the central equation that leads to $\Delta m = L/c^2$ is not applicable. Such a central equation should be free of limitations.

We also have noncompliance of the identity $a^2 - b^2 = (a + b)(a - b)$ by (1), as follows.

Further contradictory results are also self-evident if (35) is solved and the same condition ($v = c$ or $v \to c$)

is applied, i.e., $\{1 - v^2/c^2\} = (1 - v/c)(1 + v/c)$ is the simple algebraic result:

$$\ell^* = \ell \frac{\sqrt{1-v/c}}{\sqrt{1+v/c}}. \tag{35a}$$

Now again if the velocity v tends to c, i.e., $v \rightarrow c$, the above equation becomes

$$\ell^* = 0. \tag{36}$$

Under this condition, (35) in unsolved form is $\ell^* = 0/0$.

Thus the same equation (in solved and unsolved forms) under similar conditions ($v \rightarrow c$) gives different results ($\ell^* = 0/0$ and $\ell^* = 0$), which is purely arbitrary and illogical. Thus results from (1) are contradictory to the basic identity of algebra, and in addition the result is not consistent with dimensional homogeneity.

The basic principle of conservation of mass and energy should not be based on an equation that is full of limitations; e.g., it disobeys dimensional homogeneity and basic algebraic identities. Thus (1) is relativistic in nature but its numerator varies with v, even under classical conditions. The variation in magnitude of the numerator may be a factor in the inconsistent results.

If the angle $\varphi = 180°$, then $\cos 180° = -1$, so under the condition when the velocity tends to c ($v \rightarrow c$) or becomes equal to c, then

$$\ell^* = \infty.$$

Thus entirely different results are obtained,, i.e., $\ell^* = \infty$, simply if the angle of the wave is exactly reversed ($\varphi = 180°$) compared to the above case (when $\varphi = 0$, then $\ell^* = 0/0$ or $\ell = 0$).

Similarly, Einstein(16) developed the theory of the static universe in 1917, which has the limitation that Einstein divided by a term that becomes zero under certain cases. This limitation was pointed out by the cosmologist Friedmann, and later on Einstein accepted the same as the biggest blunder of his life, quoted by George Gamow.(17) A similar situation might also occur here.

Thus, in totality, critically analyzing all aspects, this derivation is obviously inconsistent and theoretically unchecked as well. As Einstein(8) did not address this aspect at all along with many others that he should have, his unfinished task is completed here.

5. EINSTEIN'S ARBITRARY GENERALIZATION OF $\Delta m = L/c^2$ FOR ALL ENERGIES; i.e., $\Delta m = \Delta E/c^2$

Further, Einstein generalized the equation from (10) in the following way in just two sentences:(8) "Here it is obviously inessential that energy taken from the body turns into radiant energy, so we are led to a more general conclusion.

"The mass of a body is a measure of its energy content: if the energy changes by L, the mass changes in the same sense by $L/9 \times 10^{20}$ if the energy is measured in ergs and mass in grams."

The meaning of the first sentence is dubious with regard to the discussion. In the second sentence Einstein(8) speculated a general equation from (10) in an analogous way as energy emitted is equal to the product of mass annihilated and c^2; i.e.,

$$\Delta m = \frac{\Delta E}{c^2} \text{ or } (M_b - M_a) = \frac{E_{final} - E_{initial}}{c^2}, \tag{37}$$

where ΔE is the energy produced on the annihilation of mass.

Now it is regarded as true for all types of energies that *the mass of a body is a measure of its energy content.* Does this mean that when sound energy is produced from a source, then the mass of the body decreases? When mass is annihilated, a huge amount of sound or noise may be produced, which can be termed as a sound bomb or a noise bomb, analogous to a nuclear bomb. The greatest likelihood of the phenomenon may be speculated at the instant of the big bang or in supernova explosions, etc. At a small scale, sound energy is produced when a bomb or a firecracker explodes. In the discussion no equation for sound energy or other forms of energy (in frames having relative motion or at rest) equivalent to (1) and (10) has been quoted, so the above speculation needs an insightful theoretical justification.

If conversion of mass to sound energy is feasible, then the speed of sound may be more than 332 m/s under some characteristic cases. The same arguments can be checked for the speed of light if mass annihilates spontaneously in bulk in some emblematic reaction and mainly light energy is produced. Such reactions may be more feasible in heavenly bodies. In the existing literature there are proposals(18,19) that the speed of light can be less or more than c. The same is true for all other forms of energy, as the law of conservation of mass-energy holds in all circumstances. In its generalization from light-energy–mass equations all aspects will be accounted for here. Do all forms of energies obey (1) or have similar dependence as in (1)? But as already discussed in Section 4, this has inherent limitations.

About its experimental confirmation, Einstein[8] said, "It is not excluded that it will prove possible to test this theory using bodies whose energy content is variable to a high degree (e.g., radium salts)." Thus at that time Einstein had in his mind the loss of weight resulting from radioactive transformations.[20] At that time, heat produced on the loss of mass in the body in burning (chemical change) was also the most obvious example. Planck[21,22] was the first to remark that the mass-energy equation bears on binding energy for a mol of water, in 1907. In 1910, Einstein himself remarked,[23] "for the moment there is no hope whatsoever for the experimental verification of mass-energy equivalence." But similar estimates for nuclear binding energy (conversion of mass to energy) were made after nearly a quarter of a century when atomic and nuclear models were developed and nuclear characteristics were understood.

For this period Einstein's mathematical derivation was not critically analyzed, and after its experimental confirmation in nuclear reactions, theoretical analysis was not felt necessary, as it is now. In nuclear phenomena the equation $\Delta E = \Delta mc^2$ is regarded as standard or reference. For the first time, in the spirit of true science, Einstein's derivation is critically analyzed.

5.1 Einstein Derived Five Equations Relating to Various Types of Masses (Rest Mass (m_0), Relativistic Mass (m_r), Mass Annihilated (Δm) and Mass Created (Δm)) and Energies (Rest Mass Energy (E_0), Relativistic Energy (E_{rel}), and Energy Annihilated or Energy Created (ΔE))

The inherent characteristics of the equations derived by Einstein are discussed below. The only closely related equation existing in Einstein's time was that for kinetic energy, i.e., $K = m_0v^2/2$:

(i) $$KE_{rel} = (m_r - m_0)c^2. \quad (12)$$

This equation is equivalent to $K = m_0v^2/2$ in relativistic mechanics.

(ii) $$E_0 = m_0c^2 \quad (38)$$

(energy when the body is at rest, $v = 0$, $dx = 0$ or $dK = Fdx = 0$). E_0 is derived from the relativistic form of kinetic energy, which is further derived from $dK = Fdx$.

(iii) $$E_{rel} = KE_{rel} + m_0c^2 = m_rc^2 \quad (39)$$

(energy when the velocity of the body is $v \sim c$). E_{rel} is derived from the work-energy theorem, i.e., work done is equal to change in kinetic energy, which is equal to KE_{final} minus $KE_{initial}$, and the relativistic variation of mass as given by (6), i.e., $m_r = \beta m_0$.

(iv) $$\Delta L = \Delta mc^2 \quad (10)$$

(conversion of light energy to mass and vice versa). ΔL is derived from energy considerations of emission of light energy in two systems having relative motion under classical conditions using the binomial theorem.

(v) $$\Delta E = \Delta mc^2 \quad (37)$$

(conversion of any form of energy to mass and vice versa). $\Delta E = \Delta mc^2$ has been speculated by Einstein from $\Delta L = \Delta mc^2$.

Now the five equations derived by Einstein are $L = \Delta mc^2$, $\Delta E = \Delta mc^2$, $KE_{rel} = (m_r - m_0)c^2$, $E_{rel} = m_rc^2$, and $E_0 = m_0c^2$. They appear to be similar as they have the same units and dimensions but conceptually are as different as the four directions. Even the origin of $E_0 = m_0c^2$ is questionable in view of scientific logic.

Similar equations of energy that are useful in understanding the conceptual differences of (10), (12), and (37)–(39) are kinetic energy (KE) and potential energy (PE):

$$KE = \frac{m_0v^2}{2}, \quad (34)$$

$$PE = m_0gh, \quad (40)$$

where g is acceleration due to gravity and h is the height at which the body is placed.

5.2 Origin and Interpretation of Rest-Mass Energy

It can be carefully noted that the equations $E_0 = m_0c^2$, $E_{rel} = m_rc^2$, $KE_{rel} = (m_r - m_0)c^2$, $KE = m_0v^2/2$, and $PE = m_0gh$ originate from the same equation, i.e.,

$$dW = dK = Fdx\cos\theta. \quad (41)$$

If the force (F) causes displacement (dx) in its own direction, then $\theta = 0°$ or $\cos 0° = 1$, so

$$dK = dW = Fdx. \quad (42)$$

Einstein gave a direct result(2) for the relativistic form of kinetic energy for the slowly accelerated electron in his Section 10 and then generalized it for all ponderable masses. In the same paper he introduced the special theory of relativity. If the velocity is in the relativistic region ($v \sim c$), then (42) becomes

$$dK = dW = Fdx = \frac{d}{dt}(m_r v)dx = m_r v dv + v^2 dm.$$

Differentiating (6), i.e., $m_r = \beta m_0$, we get

$$m_r v dv + v^2 dm = c^2 dm,$$

or

$$dK = dW = c^2 dm,$$

$$KE_{rel} = W_{rel} = c^2(m_r - m_0). \quad (12)$$

In other words, the kinetic energy attained by the body due to the influence of the external force when the body moves in the direction of the force with velocity v, which is comparable to c, the speed of light, is equal to $c^2(m_r - m_0)$, which is equal to the increase in the mass of the body due to relativistic velocity times c^2, or the work done by the body due to the influence of the external force when the body moves in the direction of the force with velocity v, which is comparable to c, the speed of light, is equal to $c^2(m_r - m_0$, which is equal to the increase in the mass of the body due to relativistic velocity times c^2.

From (12) the Newtonian form of kinetic energy can be obtained.

If m_r and m_0 are the same, then the relativistic form of kinetic energy or work done is zero.

If the velocity v is in the classical region ($v \ll c$), then (12) reduces to the classical form of KE; i.e.,

$$K = \frac{m_0}{\sqrt{1 - v^2/c^2}} - 1$$
$$= m_0\left(1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + \cdots - 1\right) = \frac{m_0 v^2}{2}. \quad (34)$$

If this condition is applied to the original (42), then the Newtonian form of the kinetic energy is obtained:

$$K = Fdx = \frac{d}{dt}\{m_0 v\}dx = \frac{m_0 v^2}{2}. \quad (34)$$

5.3 Deduction of Rest-Mass Energy by Einstein from (12)

In 1907 Einstein(3) deduced the rest-mass energy after terming $m_r c^2$ as the total energy or relativistic energy (E_{rel}), because m_r is the relativistic mass:

$$KE_{rel} = (m_r - m_0)c^2, \quad (12)$$

$$m_r c^2 = KE_{rel} + m_0 c^2. \quad (12a)$$

Einstein renamed (12a) as the relativistic energy E_{rel} or the total energy:

$$E_{rel} = m_r c^2 = KE_{rel} + m_0 c^2. \quad (39)$$

When the body is at rest, $v = 0$, $dx = 0$, or $dK = Fdx = 0$. Then Einstein wrote

$$E_{rel} \text{ (when } v = 0) = m_0 c^2 = 0 + m_0 c^2.$$

Then Einstein expressed E_{rel} (when $v = 0$) as rest-mass energy (E_0). Thus

$$E_{rel} \text{ (when } v = 0,\ dx = 0, \text{ or } dK = Fdx = 0),$$

or

$$E_0 \text{ (rest mass energy)} = m_0 c^2 = 0 + m_0 c^2. \quad (38)$$

This inference needs to be critically discussed when the body is at rest ($v = 0$, $dx = 0$), as the very first equation ($dK = Fdx = 0$) is zero and intermediate equations do not exist. Hence Einstein's rest-mass energy is a mathematically and conceptually nonexistent physical quantity, as obtained from invalid mathematical operations.

5.4 Invalidity or Inconsistencies in Einstein's Deduction, i.e., in (39)

If an equation is rearranged or renamed (simply transposing without multiplication, division, addition, or subtraction of any term), then the results from the original equation and the new transformed equation must be the same. Also, the final equation cannot be interpreted under the condition when the very first equation is zero and the preceding equations don't exist. But this simple logic is not satisfied or obeyed by Einstein's interpretation of rest-mass energy. We have

$$KE_{rel} = (m_r - m_0)c^2, \tag{12}$$

$$KE_{rel} + m_0c^2 = m_rc^2. \tag{12a}$$

$$\text{Total energy or Relativistic energy} = m_rc^2 = KE_{rel} + m_0c^2. \tag{39}$$

Now (12), (12a), and (39) are exactly the same. Thus these equations under similar conditions, e.g., when the body is at rest ($v = 0$, $dx = 0$, $dK = Fdx = 0$), must lead to similar results. Hence we substitute $v = 0$ in all three equations.

First, in (12), we have

$$0 = (m_0 - m_0)c^2 = 0.$$

Thus, under the condition $v = 0$, (12) does not give any physical information. This is because, under this condition, the first equation is $dK = Fdx = 0$, so the subsequent equations don't exist and the final equation cannot be interpreted.

Second, in (12a), we have

$$0 + m_0c^2 = m_0c^2$$

or

$$1 = 1 \text{ or } 1 - 1 = 0.$$

Thus, again, in this case, (12a) does not give any physical information, and hence cannot be interpreted regarding E_0 and m_0. The reason is exactly as above.

Third, in (39), we have

$$\text{Relativistic energy } (v = 0) = m_0c^2 = 0 + m_0c^2. \tag{43}$$

This equation can be interpreted as when the body is at rest, so the relativistic energy (dependent on velocity) is zero, as in this case kinetic energy is zero. Thus

$$0 = m_0c^2 = 0 + m_0c^2,$$

$$0 = m_0c^2 = m_0c^2.$$

m_0 and c^2 both are nonzero and their product cannot be zero. Hence, as in the previous cases under this condition ($v = 0$, $Fdx = 0$), the equation cannot be interpreted. This is further supported by (42) as

$$dK = Fdx \text{ or } dK = F \cdot 0 = 0.$$

Thus, if the very first equation and the subsequent equations do not exist, then further interpretation is not only invalid but impossible.

5.5 Einstein's Arbitrary Explanation

Without taking into account that when the body is at rest then $dK = Fdx = 0$, Einstein tried to interpret the equation in such a way that the result may resemble (10) or (37), i.e.,

$$\Delta L = \Delta mc^2 \text{ or } \Delta E = \Delta mc^2,$$

$$\text{Relativistic energy } (v = 0,\ dx = 0,\ \text{or } dK = Fdx = 0\}$$
$$= \text{Rest mass energy, } E_0.$$

Thus (39) becomes

$$\text{Relativistic energy } \{v = 0, Fdx = 0\} E_0$$
$$= 0 + m_0c^2 = m_0c^2$$

or

$$E_0 = m_0c^2 = m_0c^2.$$

Einstein interpreted this equation as the relation between rest mass, m_0, and rest-mass energy, E_0. In the process the following facts were ignored.

(i) When the body is at rest, the very first equation vanishes, i.e.,

$$dK = dW = Fdx = 0,$$

and all other equations are nonexistent. Thus to draw any conclusion from the imaginary equations is purely *hypothetical* and has no logical or mathematical basis.

(ii) Equations (12) and (12a) are originating forms of (39). Under similar conditions, the results from (12) and (12a) are the same, but the results from (39), which is just another form of (12) and (12a), are different. Hence (39) may be regarded as illogically renamed.

5.6 Δm in the Equation $\Delta m = \Delta L/c^2$ or $\Delta m = \Delta E/c^2$ Is Different from m_0 in $E_0 = m_0c^2$, $KE_{rel} = (m_r - m_0)c^2$, $KE = m_0v^2/2$, $U = m_0gh$, and $p = m_0V$

This conclusion is justified by the following illustrations.

5.6.1 Origin and Meaning of Δm

Of the above equations only (10) and (37) represent mass annihilated to energy or energy materialized to mass. Here Δm is mass actually annihilated to energy.

For the hydrogen isotope deuterium,(7) the expected mass is 2.0165 a.m.u.; i.e.,

$$\begin{aligned}&\text{Mass of } {}_1\text{H}^1 \text{ atom} + \text{Mass of neutron}\\&= 1.0078 \text{ a.m.u.} + 1.0087 \text{ a.m.u.} = 2.0165 \text{ a.m.u.},\end{aligned} \tag{44}$$

$$\begin{aligned}&\text{Experimentally measured mass of deuterium}\\&= 2.0141 \text{ a.m.u.},\end{aligned} \tag{45}$$

$$\begin{aligned}&\text{Mass defect} = \text{Expected mass} - \text{measured mass}\\&= 0.0024 \text{ a.m.u.},\end{aligned} \tag{46}$$

$$\begin{aligned}&\text{Energy equivalent to mass defect}\\&= (0.0024 \text{ a.m.u.}) \times 931 \text{ MeV/a.m.u.}\\&= 2.2 \text{ MeV}.\end{aligned} \tag{47}$$

Here Δm is the mass defect 0.0024 a.m.u., i.e., the mass that is actually converted to energy. Hence Δm is not equal to the actual mass of deuterium, i.e., 2.0165 a.m.u. Thus here Δm (0.0024 a.m.u.) is given in (10) or (37), i.e., $\Delta m = \Delta L/c^2$ or $\Delta m = \Delta E/c^2$, rather than m_0 in the equation $m_0 = E_0/c^2$.

Einstein has considered in his derivation that the mass of the body decreases when light energy is emitted and is given by

$$\frac{M_b v^2}{2} - \frac{M_a v^2}{2} = \frac{L v^2}{2c^2}. \tag{11}$$

In other words, the mass of the body before emission (M_b) minus the mass of the body after emission (M_a) is equal to L/c^2.

Analogously, the above equation for the deuterium binding energy can be written as

$$\text{Mass of free nucleons} - \text{Mass of bound nucleons} = \frac{\text{Binding energy (B.E.)}}{c^2},$$

$$\Delta m\,(M_b - M_a) = \frac{\Delta L}{c^2} = \frac{\text{B.E.}}{c^2}. \tag{10}$$

Then from (10) the general form of the mass-energy equation, i.e., $\Delta E = \Delta mc^2$, is speculated.

5.6.2 The Meaning of m_0 Is the Same in All Equations

The rest mass m_0 is related to the relativistic mass m_r as

$$m_r = \frac{m_0}{\sqrt{1 - v^2/c^2}}. \tag{6}$$

This equation existed before Einstein's special theory of relativity and was used by him in the same.(2) Kaufmann(9) verified it in 1901, as did Bucherer(10) more comprehensively in 1908. According to this, if the body is at rest or moving with slow velocity, then

$$\text{Relativistic mass} = \text{Rest mass},$$
$$\text{i.e., } m_r = m_0.$$

The rest-mass term occurs in the equations $E_0 = m_0c^2$, $KE_{rel} = (m_r - m_0)c^2$, $KE = m_0v^2/2$, and $PE = m_0gh$. Now the equation $E_0 = m_0c^2$ cannot be interpreted as energy E_0 produced on the annihilation of rest mass m_0, or when energy E_0 is materialized then rest mass m_0 is produced, as these are not defined for that purpose. If so, then it is equally feasible to interpret that when m_0 is annihilated, KE and PE must both increase and must be given by the expressions $m_0v^2/2$ and m_0gh, respectively. In these terms, c is not involved. The same argument is valid for $KE_{rel} = (m_r - m_0)c^2$. Can a similar explanation be postulated for momentum $p = m_0v$ (defined under classical conditions)? Each equation is derived under characteristic conditions and is applicable under those conditions only. Hence the annihilation of mass to energy or the materialization of energy to mass is only explained on the basis of (10) or its speculative generalization, i.e., $\Delta E = Ac^2\Delta M$.

6. A NEW OR GENERALIZED FORM OF MASS-ENERGY EQUIVALENCE AS $\Delta E = Ac^2\Delta M$

In view of the serious mathematical limitations of Einstein's derivation, the mass-energy equation is derived in independent ways. Here we start with a new method that is nonrelativistic in nature (as (6) is not involved), as mass can be annihilated to energy when the velocities of the reactants are in the classical region. This aspect is justified, because in the fission of ${}_{92}U^{235}$, secondary neutrons produced originally have energy 1–2 MeV (velocity in relativistic region). This is reduced to 0.025 MeV (2185 m/s) with the help of a moderator (such as heavy water); otherwise nuclear fission does not take place. This velocity is in the classical region and even the orbital velocity of Earth (3×10^4 m/s) is in the classical region. Some anomalies are observed in nuclear fission fragments of U^{235} and Pu^{239}, as the total kinetic energy (TKE) observed experimentally[(24–26)] is 20–60 MeV less, which is explained on the basis of the wave mechanics equation $H = mv^2$, obviously given by de Broglie from his wave mechanics. Alternatively, the liquid-drop model of Bohr and Wheeler can also be extended[(27,28)] to explain the deficiency.

Interestingly, the derivation of Einstein's equation (10) started from the relativistic equation, i.e., (1), but he finally derived the results classically by applying the binomial theorem as cited above. In the whole derivation, the relativistic variation of mass, i.e., $m_r = \beta m_0$, as given by (6), is not taken into account. Further, scientists are continuously suggesting variations (increase or decrease) in magnitudes of c.[(18,19)] If these variations occur, then this derivation of $\Delta E = \Delta mc^2$ will become invalid, or energy emitted will be less or more than predicted by Einstein's equation. Then existing phenomena cannot be explained with the help of Einstein's $\Delta E = \Delta mc^2$.

In the 18th century Lavoisier stated the law of conservation of matter in chemical reactions. The first idea of the mass-energy interconvertibility version was given by Fritz Hasenohrl[(1)] in 1904. However, there are a few scientifically unconfirmed counterclaims about it ($\Delta E = \Delta mc^2$). In 1905 Einstein mathematically derived the interconvertibility equation between mass and energy as $\Delta L = \Delta mc^2$. According to this, the conversion factor between mass and energy is precisely and rigidly c^2. Whereas, according to $\Delta E = Ac^2\Delta M$, the conversion factor between mass and energy may or may not be c^2.

Here the derivation involves the calculation of an infinitesimally small amount of energy dE when a small amount of mass dm is converted (in any process) into energy (energy may be in any form, e.g., light energy, sound energy, energy in the form of invisible radiation, etc.). Then

$$dE \propto dm.$$

In the existing literature the conversion factor c^2 between mass and energy has been experimentally confirmed. Thus, in the above proportionality, it can be taken into account as

$$dE \propto c^2 dm \text{ or } dE = Ac^2 dm, \tag{48}$$

where A is used to remove the sign of proportionality and has a nature like Hubble's constant (50–80 km/s-Mpc), the coefficient of viscosity (1.05×10^{-3}–19.2×10^{-6} poise), the coefficient of thermal conductivity (0.02–400 $Wm^{-1}K^{-1}$, etc.), or the decay constant λ in radioactivity; the force or spring constant k is another similar example. It may be called the conversion coefficient as it highlights the extent of the conversion of mass to energy or vice versa and depends on the characteristics and intrinsic conditions of a particular process.

Further, the conversion coefficient A is consistent with Newton's second law of motion or Axiom II as quoted in Book I of the *Principia* in Latin (first translation in English by Andrew Motte, 1729; subsequent disseminations have directly used acceleration): "The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed."

Mathematically,

$$F \propto a \text{ or } F = m_0 a, \tag{49}$$

where m_0 (mass) is a constant of proportionality that varies from one situation to another, similar in nature to A (conversion coefficient) or Hubble's constant, etc. Likewise, in the law of gravitation, G is constant.

Let the mass decrease from M_i to M_f and the energy increase from E_i to E_f in some conversion process. Initially, when no mass is converted into energy, $E_i = 0$. Thus, integrating (48), we get

$$E_f - E_i = Ac^2(M_f - M_i), \tag{50}$$

$$\Delta E = Ac^2\Delta M. \tag{51}$$

In other words, the energy evolved is equal to Ac^2 (decrease in mass).

If the initial and final masses remain the same, i.e., $M_i = M_f$, then ΔE, the energy evolved, is zero.

If the characteristic conditions of the process permit and whole mass is converted into energy, i.e., $M_f = 0$, then (50) becomes

$$\Delta E = -Ac^2 M_i. \tag{52}$$

Here energy evolved is negative, which implies that energy is created at the cost of mass, and the reaction is exothermic in nature. Also, energy is a scalar quantity, hence only magnitude is associated with it and not direction. In the case of the annihilation of an electron-positron pair, all the mass is converted into energy, i.e., $M_f = 0$, so the energy emitted is consistent with Einstein's mass-energy equivalence, i.e., $\Delta E = Ac^2\Delta M$. Thus in this case the value of A is unity, so (50) becomes

$$\Delta E = -c^2\Delta M = -c^2 M_i. \tag{53}$$

If ΔE is positive, the additional mass supplied to the system of M_f is more than M_i, but in (50) the annihilation of mass into energy is being discussed, so this case is irrelevant.

Further, (50) implies that if the initial and final energies are equal ($E_f = E_i$), then no mass is annihilated. The created mass is maximum if all the energy is materialized into mass, i.e., $E_f = 0$ (e.g., materialization of gamma-ray photon).

Thus mass-energy equivalence may be stated as follows: "The mass can be converted into energy or vice versa under some characteristic conditions of the process, but the conversion factor may or may not always be c^2 (9×10^{16} m^2/s^2) or c^{-2}."

6.1 Equation (51) Can Be Obtained by the Method of Dimensions

Let the energy emitted (ΔE) on the annihilation of mass depend on the annihilated mass (ΔM) as a power of a, the speed of light c as a power of b, and time t as a power of c. Thus, as in other cases in physics, ΔE can be expressed as

$$\begin{aligned} \Delta E &\propto (\Delta M)^a c^b t^c, \\ \Delta E &= A(\Delta M)^a c^b t^c, \end{aligned} \tag{54}$$

where A is a constant of proportionality and is called the conversion coefficient. According to the principle of dimensional homogeneity, the dimensions of both sides must be the same. Hence

$$ML^2T^{-2} = AM^a(LT^{-1})^b T^c = AM^a L^b T^{-b+c},$$

$$\text{or } a = 1,\ b = 2, \text{ and } -2 = -2 + c \text{ or } c = 0,$$

or

$$\Delta E = A\Delta M c^2 t^0 = Ac^2\Delta M. \tag{51}$$

Hence the same result is obtained by the method of dimensions.

6.2 The Variation in Magnitude of *A* Is Consistent with Existing Physics

Now the obvious question is how should the coefficient A vary; i.e., what factors is it dependent on or influenced by? The answer to the question of dependence or variation of the coefficient of proportionality A is precisely the same as the answer for all other proportionality constants or coefficients in existing physics. All such constants or coefficients of proportionality in existing physics depend on the intrinsic characteristics, conditions, and parameters that influence the results directly or indirectly; hence the same is precisely true for A. The coefficient A is dimensionless because it is introduced in the existing equation of energy, and the dimensions of energy have to be ML^2T^{-2}, the same on both sides, so, in $F = kma$, k is also dimensionless.

For example, Hubble's constant (50–80 km/s-Mpc), the coefficient of thermal conductivity (0.02–400 Wm^{-1}K^{-1}), the coefficient of viscosity (1.05×10^{-3}–19.2×10^{-6} poise), the decay constant, etc., all are determined experimentally. All these vary from one situation to other. The value of G in various experiments appears to be higher than the current accepted value (Gundlach et al.[29]).

For example, $_{92}U^{235}$ undergoes nuclear fission but not $_{26}Fe^{57}$. Further, for nuclear fission to take place, in a chain reaction fast-moving neutrons (energy 1–2 MeV, velocity in relativistic region) are produced; these are slowed down and called thermal neutrons (0.025 MeV, velocity 2185 m/s) with the help of a moderator. The TKE of the fission fragments of U^{235} or Pu^{239} is found experimentally to be 20–60 MeV less than the Q value (200 MeV) predicted by Δmc^2. Whereas an intrinsic condition for nuclear fusion is that the temperature be of the order of 10^6 K, the feasible conditions to cause fission of $_{26}Fe^{56}$ have not yet been achieved. Such characteristic conditions are taken into account by the constant of proportionality. This aspect is established in existing physics, and in the generalized form of the mass-energy equation the same is taken into account by A. Energy may be in different forms, e.g., heat energy, light energy, sound energy, electrical energy, chemical energy, atomic energy, etc., under different conditions. Or, in some particular process, the energy may coexist in different forms under different conditions; e.g., in the explosion of a nuclear bomb, heat, sound, light, and energy in the form of invisible radiation are released simultaneously. Thus, along with the characteristics and intrinsic conditions of the processes, the value of A varies from unity.

The constant of proportionality may arise by the method of conceptual derivation or by the method of dimensions, but it is always determined experimentally. In physics the same entity may behave in different ways under different conditions. For example, a single wave of radiation behaves like both a wave and a particle; also, an atomic particle electron behaves like both a wave and a particle, depending on the characteristic conditions. Thus the status of the conversion coefficient A and its magnitude is consistent with existing physics.

6.2.1 What Determines A?

The value of A is determined by the magnitudes of mass annihilated to energy or energy materialized to mass., i.e., $A = \Delta E/c^2\Delta M$. The following two questions are important in understanding the determination of A:

(i) The mass of which particular element will be annihilated? If mass is annihilated, then what is the magnitude of the mass annihilated (ΔM) out of the total mass M? Both of these aspects depend on the inherent characteristics of the elements.

(ii) Which factor determines (for the same annihilated mass) whether the value of A will be less than, greater than, or equal to one?

The value of A depends on the inherent characteristics of the elements. To determine the magnitude and trends of variations of A, all the above reactions must be extensively theoretically and experimentally conducted. The general trend of the variation of A and its magnitude is expected to be different in each type of reaction, due to different inherent conditions prevailing in nuclear reactions, chemical reactions, heavenly processes, etc. Thus, due to the diverse conditions of the processes, the mathematical formula for the magnitude and variation of A in $\Delta E = Ac^2\Delta M$ can only be developed after extensive experimentation over a wide range of parameters. This is supported as below.

6.2.2 Determinations of Hubble's Constant H and A Are Equivalent

This determination of A is consistent with the determination of Hubble's constant H, which is given by $H = V/D$. Now, substituting the values of V (velocity of recession) and D (distance), Hubble's constant can be measured. The determination of A is also identical, as it is determined after the measurement of ΔE and ΔM. The range of variation of Hubble's constant experimentally measured is 50–80 km/s-Mpc; likewise A will have its own characteristic range. (The Hubble constant H may be better called the Hubble coefficient due to its variation in magnitude.)

6.2.3 Resistance R in Ohm's Law

Ohm's law establishes the relationship between electromotive force E, current I, and resistance R; i.e., $E \propto I$, or

$$E = RI = \left[\frac{\rho l}{a}\right] I = \left[\frac{m_e l}{ne^2\tau a}\right] I = \left[\rho_0 \exp(E_g / kT)\frac{l}{a}\right] I, \quad (55)$$

where m_e is the mass of the electron, n is the number of electrons per unit volume, τ is the relaxation time, l is the length, and a is the area of the conductor. Thus the resistance R, which is a constant of proportionality in Ohm's law, is further determined as in (4), as there are direct experimental observations. This type of determination as in (4) of A is not possible due to the lack of experimental data as described above.

Further, the equation $\Delta E = Ac^2\Delta M$ involves all types of reactions, so separate experiments are required for the determination of A variation and the trend of A in each case.

7. CONVERSION OF ENERGY FROM ONE FORM TO ANOTHER

In an electric bulb, electrical energy changes to light energy; in a radio, electrical energy is converted into sound energy; in a cell, chemical energy is changed to electrical energy; in a photocell, light energy changes to electrical energy; and so on. There are many such examples of interconversion of one form of energy to other. How much mass is associated with these interconversions of energy from one form to other? If these are pure examples of interconversion of energy from one form to another, the energy in one form may be regarded as proportional to that in the other; e.g.,

$$\text{Light energy} = k \times \text{Electrical energy}, \quad (56)$$

where k is a coefficient that determines the characteristics of the conversion of light energy to electrical energy. Equation (56) is similar to the relation between heat and work ($W = JH$, $J = 4.2 \times 10^7$ erg cal^{-1}). Likewise, similar equations can be written for other forms of energy and can be verified.

8. UNSTUDIED ASPECTS OF $E = mc^2$ AND APPLICATIONS OF $\Delta E = Ac^2\Delta M$

The experimental confirmation of $\Delta E = \Delta mc^2$ came in the 1920s after two decades of speculation, realistically at the dawn of the experimental nuclear physics era. Then it was regarded as the standard of measurement of energy such as mega-electron volts (MeV), and all the observations were based on it. Then qualitative nuclear uncontrolled fission established it further, so it was not felt necessary to theoretically analyze it. It rigidly establishes the conversion factor between mass and energy as c^2. Even then there was some compelling evidence that $E = mc^2$ is not complete, but this evidence was either overlooked or no serious attempt was made to pursue that direction, possibly due to the lack of a viable alternative such as $\Delta E = Ac^2\Delta M$.

8.1 $\Delta E = c^2\Delta M$ Is Unconfirmed in Chemical Reactions

Further, $\Delta E = \Delta mc^2$ has not been confirmed in the most abundant chemical reactions; the reason cited is that currently experimental precision is not accurate enough to measure the mass annihilated and energy emitted.[7,31,32] Purposely, a reaction from the existing literature is cited as quoted by Halliday.[32] Suppose that 1.0 mol of (diatomic) oxygen interacts with 2.0 mol of (diatomic) hydrogen to produce 2.0 mol of water vapor, according to the reaction

$$2H_2 + O_2 = 2H_2O + Q\,(4.85 \times 10^5 \text{ J}).$$

The energy released in the reaction is 4.85×10^5 J. According to $\Delta E = \Delta mc^2$, this energy is equivalent to the mass 5.39×10^{-12} kg (or 5.92×10^{18} electronic masses). Thus the mass of the reactants decreases and is converted into energy, and the mass is a fraction of electronic masses. This means that hydrogen and oxygen (hence the electrons, protons, and neutrons) become lighter by the mass 5.39×10^{-12} kg.

Or does it mean that masses of the order of a fraction of an electron are possible (for transference, annihilation, or creation), as charges of the order of fractions of electronic charge are considered in the quark concept? If so, then this deduction is consistent with the proposal that electrons consist of subparticles. So all possibilities have to be discussed. But at the moment, a lack of experimental precision keeps this as speculative only. But in this regard the key factor (to answer every query) is precise measurement of annihilated mass, which has not been done yet.

Apparently, there is an immense conceptual difference between the annihilation of an electron-positron pair in nuclear reaction and in combustion (e.g., burning of paper), as described above. So these experiments are required to be specifically conducted. If emitted energy is found to be less than Δmc^2, then the value of A is less than one, which is an open prospect so far.

8.2 TKE of Fission Fragments Is 20–60 MeV Less than Predicted by $\Delta E = c^2\Delta M$

In the laboratory it has been confirmed[24–26] that using thermal neutrons the TKE of fission fragments that result from U^{235} and Pu^{239} is 20–60 MeV less than the Q value of the reaction predicted by Einstein's famous $\Delta E = \Delta mc^2$ (200 MeV for U^{235}). It is typically assumed to explain it that energy (Q value minus TKE of fragments) is lost in unobserved effects.

Attempts have been made to explain the Q value of the reaction on the basis of de Broglie's wave-mechanical (8), i.e.,

$$H = mv^2, \quad (57)$$

where H is energy, m is mass, and v is velocity ($v < c$). Also, attempts[27,28] have been made to explain the TKE of fission fragments by extending the successful liquid-drop model of Bohr and Wheeler. These attempts use the fact that TKE is the Coulomb potential energy equation and, in a specific case (TKE should have minimum value), the magnitudes of TKE as given by the extended model coincide with the wave-mechanical equation $H = \Delta mv^2$. The theme of this discussion is that extension of the Bohr–Wheeler model yields the same value of TKE[24] as $H = \Delta mv^2$; further, the equation $\Delta E = Ac^2\Delta M$ is consistent with $H = \Delta mv^2$ with a value of A less than unity. Let the TKE of fission fragments be 175 MeV (as experimentally it is observed to be less), instead of the expected 200 MeV. Then, according to $\Delta E = Ac^2\Delta M$, the value of A

is 0.875; i.e.,

$$A = \frac{\Delta E}{c^2 \Delta M} = \frac{175}{200} = 0.875. \qquad (58)$$

Thus the energy of the fission fragments of U^{235} and Pu^{239} is given by

$$\Delta E = 0.875 c^2 \Delta M. \qquad (59)$$

Thus the energy obtained corresponding to the annihilation of mass ΔM is less, so the TKE of nuclear fragments is less. This problem occurs because, according to $\Delta E = \Delta mc^2$, the conversion factor between ΔE and Δm for all cases is strictly c^2, but this is not always true, as indicated by $\Delta E = Ac^2 \Delta M$.

8.3 Discovery of Particle with Mass Less than the Predicted Mass

Recent work at SLAC confirmed the discovery of a new particle dubbed Ds (2317) with mass 2317 MeV. But this mass is far less than current estimates, which is a mathematical puzzle.[30] This discrepancy can be explained with the help of the equation $\Delta E = Ac^2 \Delta M$ with a value of A more than one.

8.4 Applications of $\Delta E = Ac^2 \Delta M$ in Cosmological and Related Phenomena

For all such phenomena that involve interconversion of mass and energy, $\Delta E = Ac^2 \Delta M$ provides a reasonably good explanation with the value of A less than or greater than one. To determine A, the value of ΔM, i.e., the mass annihilated in the case of a heavenly body, is required, which cannot be directly measured like many other parameters. Thus, initially, for simplicity of calibration (the standard of reference can be chosen), the magnitude of ΔM is regarded as 4.322×10^9 kg, i.e., the mass annihilated in the case of the Sun (the luminosity of the Sun is 3.89×10^{26} Js^{-1}). Thus

$$\Delta M = \frac{\Delta E}{c^2} = \frac{3.89 \times 10^{26}\ Js^{-1}}{9 \times 10^{16}\ m^2 s^{-2}} = 4.322 \times 10^9\ kg. \qquad (60)$$

If for some cases the value of ΔM is experimentally measured, then its actual value (ΔM) can be used instead of (60). Many more such phenomena may be revealed as the innovative precision in investigative measurements increases.

8.4.1 Creation of Mass of Universe (10^{55} kg) Before the Big Bang

The big-bang theory assumes that initially ($t = 0$) the whole mass (10^{55} kg) of the universe was infinitely compact and in a singular state enclosing a space even smaller than an atomic particle. Then it instantaneously exploded in a gigantic detonation and has been expanding ever since.[33] How was the whole mass of the universe formed and condensed to an infinitely compact point? How was this compressional energy created and how did the whole mass become hot? How was the explosion triggered, causing expansion and dramatic reduction in temperature and density? Which source provided the energy for these events? None of these questions are answered by either detractors or adherents of the big-bang theory and are open for plausible elucidation.

Further, additional energy (which may be infinitely large, i.e., too high to be measured) is required to change a mass of 10^{55} kg *into a point of exceedingly high density*, and *raise the temperature*, trigger an explosion, and impart *kinetic energy* to it, so that even now it is accelerating outward. This additional energy may be far more than the energy required for creation of a mass of 10^{55} kg.

8.4.2 Creation of Mass of 10^{55} kg on the Basis of $\Delta E = \Delta Mc^2$

A gamma-ray photon of energy at least 1.02 MeV (1.623×10^{-13} J) that passes near the nucleus gives rise to an electron-positron pair of mass 18.2×10^{-31} kg; this is consistent with $\Delta E = \Delta mc^2$. The mass of the universe is estimated to be nearly 10^{55} kg, thus as above it must have been materialized from energy ($\Delta E = \Delta mc^2$) of 9×10^{71} J. Now it has to be assumed that energy 9×10^{71} J and a spectacular amount of additional energy as mentioned above was created from nothing automatically and spontaneously. The law of conservation of energy does not permit creation of mass out of nothing at all (especially on such a great scale), hence the law was not obeyed at that stage, according to $\Delta E = \Delta mc^2$. How was the energy of the order of 9×10^{71} J produced along with additional energy? Thus the answer to a question is another question.

Consider the creation of the mass of 10^{55} kg on the basis of $\Delta E = Ac^2 \Delta M$. Thus the equation $\Delta E = Ac^2 \Delta M$ predicts that in this primordial bang (an exceptionally special event), a diminishingly small pulse of energy, say 10^{-4444} J (or less), equivalent to 2.4×10^{-4443} cal (or less), can manifest itself in a mass of 10^{55} kg if the value of A is regarded as 2.568×10^{-4471}. The energy

10^{-4444} J or less is regarded to have existed inherently in the universe when the process of the formation of space started, i.e., even when there was no material particle. The primordial value of the conversion coefficient A_{uni} (due to the lack of a value of ΔM_{uni}, (60) is used) can be estimated as

$$A_{uni} = \frac{10^{-4444}}{9\times10^{16}\times4.32\times10^{9}} = 2.568\times10^{-4471}. \quad (61)$$

Thus $\Delta E = Ac^2\Delta M$ is the first equation that at least theoretically predicts that the universe (10^{55} kg) was created from an immeasurably small amount of energy (10^{-4444} J or less, which may be easily available compared to 9×10^{71} J). Thus the generalized equation explains the origin of the mass of the universe with ease.

8.4.3 Big Bang, How and Why

Most peculiar phenomena, such as the big bang, require peculiar perceptions to be understood. Here the questions are how the whole mass of the universe (already formed) condensed to a dense point with exceptionally high temperature and how the explosion was triggered. Aspects like the formation of primeval mass can be easily explained on the basis of $\Delta E = Ac^2\Delta m$ and generalization of established facts. The manifestation of energy from one form to another is well established ($H = W/J$, i.e., heat energy is converted into mechanical energy, $J = 4.2 \times 10^7$ erg cal^{-1}). The analogous relation between energy emitted on annihilation of mass and gravitational energy (measure of gravitational force or pull) is

$$\text{Energy emitted in annihilation of mass } (Ac^2\Delta m) = k \times \text{Gravitational energy } (U_g),$$

or

$$\text{Gravitational energy } (U_g) = \text{Energy emitted in annihilation of mass } (Ac^2\Delta m)/k \quad (62)$$

Consider the annihilation of mass to gravitational energy. It is experimentally observed in uncontrolled nuclear fission that light energy, heat energy, sound energy, and energy in the form of invisible radiation are emitted. This implies that energy changes from one form to another, i.e., heat energy to sound energy or light energy, etc.

The mass may first change to energy (including heat energy), then the energy may further transform to gravitational energy like heat energy changes to work. Thus it can be speculated that the earliest mass was converted into energy according to $\Delta E = Ac^2\Delta m$ with an exceptionally high value of A, thus corresponding to the case when with the annihilation of a small mass a huge amount of energy is emitted. This energy got transformed into gravitational energy as given by (62), where the smaller the value of k, the higher the congregation of gravitational energy. Hence when the whole mass of the universe was condensed to a point, the value of A in $\Delta E = Ac^2\Delta m$ was exceptionally large and the value of k was exceptionally small, so the gravitational energy and temperature of the universe were unimaginably high. The gravitational energy is universally prevalent and is an inherent property of bodies, and here it can be understood with ease how it came into being with the help of $\Delta E = Ac^2\Delta m$ and (62). Thus gravitational energy is another form of mass, as it is obtained on the annihilation of mass just like heat, light, sound energy, etc. (these energies can be physically felt). Also, the higher the mass of the body, the higher the gravitational force, gravitational potential energy, and gravitational potential.

The body with higher or maximum gravitational energy or force attracted other constituents toward its center. When lighter bodies stuck to the heavy body (under extreme conditions of temperature and gravitation), the gravitational force of the system increased. Thus high temperature and high gravitational pull caused the universe to contract to a single point; further, some other effects might have facilitated the same. As the process of annihilation of mass continued, the rise in temperature and increase in gravitational energy continued. Thus bodies were quickly attracted and, being extremely hot, they compressed to a small size easily, so the radius of the universe decreased. This process of increase in gravitational energy continued with annihilation of mass, causing compression of mass to an optimum limit. When the gravitational energy further increased on annihilation of mass and tried to compress the mass of the universe beyond the optimum limit, then it changed into a repulsive or antigravitational force, which caused the disruptive big bang.

During the bang, a considerable amount of the mass of the universe was annihilated to energy, where the value of A was much greater than one. The energy obtained after the annihilation of mass transformed into the kinetic energy of constituents, so they are continuously receding even now.

Now we compare the antigravitational force with the intermolecular force. When the size of the universe decreased beyond the optimum limit, the attractive force suddenly changed into an antigravitational force. This repulsive force, also known as dark energy, caused the explosion and recession between various constituents of the universe. This transformation of gravitational to antigravitational energy is also consistent or identical with well-established intermolecular potential energy (intermolecular force). At a larger distance compared to r_0 (equilibrium distance), the potential energy is negative, e.g., the attractive force between various molecules (like gravitation), and when the distance between molecules becomes less than r_0, the potential energy is positive, i.e., the force between molecules becomes repulsive (like antigravitational force). Nuclear force is also repulsive.

Mass and gravitation are inherently related. In the nucleus the binding energy is also due to the annihilation of a part of the mass of nucleons: the higher the mass annihilated, the higher the binding energy according to $\Delta E = \Delta mc^2$. Analogously, it can be concluded or speculated that gravitational energy has a similar nature: the higher the mass annihilated, the higher the gravitational energy according to $\Delta E = Ac^2\Delta M$. Mathematically, the gravitational force or energy of the body is proportional to the mass of the body and it emerged (another form of mass of the body) when the mass was annihilated. Hence they are inherently associated with each other. In general, the higher the bulk mass of the body and the higher the mass annihilated, the higher the gravitational energy, as in (62). The gravitational energy created is proportional to the mass annihilated.

This may be compared with magnetizing an iron bar by rubbing a magnet on it. Gradually, the degree of magnetization increases, as in the case of gravitational energy. All the heavenly bodies inherited characteristics of gravitation when the mass formed and the whole mass of the universe condensed at a point. There may be typical heavenly bodies in the universe that have gravitation just to keep their mass together and not to be able to attract other external bodies to their surfaces.

Every perception of the origin of the universe also requires an assessment about the possible end. The view of current observations that all heavenly bodies are accelerating outward implies that the universe is in initial stages as accelerating due to the impact of energy that was obtained during the big bang. At the final stages this acceleration will slow down and may become zero. At the final stages the whole mass of the universe may further convert into energy according to $\Delta E = Ac^2\Delta m$ (with the value of A less than one). Thus the whole mass of the universe may convert into a small amount of energy. Then the mass of the universe will again be created according to $\Delta E = Ac^2\Delta m$ (value of A less than one) from the small energy left from the previous cycle. This whole process may take an infinitely long time to complete, and these cycles of mass-energy interconversion may continue indefinitely.

8.4.4 Annihilation of Antimatter in Hadron Epoch

The universe began with the explosion of a "primeval atom." The uncertainty principle of quantum mechanics prevents our speculating on times shorter than 10^{-43} s after the big bang. After time 10^{-35} s in the universe, roughly equal amounts of matter and antimatter were created. Now the mass of the universe is regarded as 10^{55} kg. But so far, no antimatter domains have been detected in space within 20 Mpc of Earth.[34] The antimatter was annihilated at 10^{-6} s, at the end of the hadron epoch (10^{-35}–10^{-4} s, temperature 10^{27}–10^{12} K), and the temperature of the universe in the next lepton epoch (10^{-4}–10^{2} s) reduced to the temperature 10^{12}–10^{9} K.

This leaves a ticklish situation that has been overlooked. If a huge amount of antimatter was instantly annihilated, a huge amount of energy according to $\Delta E = c^2\Delta m$ would have been created, further increasing the temperature. But almost simultaneously the temperature of the universe decreased. The generalized equation $\Delta E = Ac^2\Delta m$ can be used to explain this, with a value of A much less than one. Thus at the end of the hadron epoch a huge amount of antimatter was annihilated and a very small amount of energy was emitted. Thus $\Delta E = Ac^2\Delta m$ (with a value of A less than one) explains both these aspects, i.e., why antimatter is not observable now and why the temperature of the universe decreased.

8.4.5 Gamma-Ray Bursts

Gamma-ray bursts (GRBs) are intense and short (approximately 0.1–100 s long) bursts of gamma-ray radiation that occur all over the sky approximately once per day and originate at very distant galaxies (several billion light-years away). GRBs are the most energetic events after the big bang in the universe and the energy emitted is approximately 10^{45} J, with the most extreme bursts releasing up to 10^{47} J. This energy cannot be explained with $\Delta E = c^2\Delta m$ (precisely confirmed in nuclear reactions). This is also the amount of energy released by 1000 stars like the Sun over their entire lifetime! This implies that for the annihilation of dwindling mass in a short time, an unimaginably high amount of energy is emitted, which can be explained with the help of $\Delta E = Ac^2\Delta M$ with an exceptionally high value of A. If for simplicity the value of ΔM can be taken standard as in (60), as an actual estimate of ΔM for GRBs is not available, then

$$A_{grb} = \frac{\Delta E}{c^2\Delta M} = \frac{10^{45}}{9\times10^{16}\times(4.32\times10^9)} = 2.57\times10^{18} \quad (63)$$

or

$$\Delta E = 2.57\times10^{18}c^2\Delta M. \quad (64)$$

Hence not all conversions of mass to energy in nature are according to $\Delta E = c^2\Delta m$, where c is the conversion factor like a universal constant. In the GRBs, intense and short bursts of gamma-ray radiation are emitted, which implies that for a small mass (simply gamma rays), in a small region, in a small time, a huge amount of energy is liberated. This is a direct confirmation for $\Delta E = Ac^2\Delta M$ with a very high value of A; i.e., for the annihilation of a small mass (GRB), in a short time, an enormous amount of energy is emitted (in this case 2.31×10^{32} J for the annihilation of 10^{-3} kg), which is 2.57×10^{18} times more than $\Delta E = c^2\Delta m$. However, the actual value of A_{grb} will be more when exact values of Δm corresponding to the energy emitted are experimentally determined, instead of the standard value as given by (63).

8.4.6 Quasars

The observations taken with the 2.5 m Isaac Newton Telescope at La Palma in the Canary Islands reveal that a quasar is 4×10^{15} (15.56×10^{41} Js^{-1}) to 5×10^{15} times brighter than the Sun, or this energy is a thousand times more than emitted by the brightest galaxy. The most peculiar characteristic of quasars as reported by Arav[35] is that this prodigious amount of energy is generated in a small region approximately *one light-year* across. By comparison, the diameter of the Milky Way is about *100 000 light-years*. This implies that, corresponding to a small region (a measure of mass and hence its annihilation), a mammoth amount of energy is emitted in the case of quasars. $\Delta E = Ac^2\Delta M$ is useful in explaining such aspects. Now

$$A_{qu} = \frac{\Delta E}{c^2\Delta M} = \frac{15.56\times10^{41}\ \mathrm{Js}^{-1}}{9\times10^{16}\times(4.32\times10^9)} = 4\times10^{16}. \quad (65)$$

For simplicity, the value of ΔM is taken as in (65), which is the standard for the sake of calculations when an exact estimate of ΔM is not available.

With this value of the generalized mass-energy, the interconvertibility equation becomes

$$\Delta E_{qu} = 4\times10^{16}c^2\Delta M. \quad (66)$$

Thus, corresponding to a small mass (size), the energy emitted is more, so comparatively smaller quasars or in general smaller bright objects are feasible. So in a small region, even when a small amount of mass is annihilated, a huge amount of energy is emitted. The lower limit of a quasar's mass has not yet been determined.[36] This is further justified by the fact that the quasars may possibly or inexorably end as super-massive black holes; currently the maximum mass is of the order of 2×10^{40} kg.[37] Thus, in spite of emitting a huge amount of energy in its own lifetime, the quasar still has a significant amount of matter, which is expected to behave like a super-massive black hole. This aspect is easily explained on the basis of $\Delta E = Ac^2\Delta M$, with a high value of the conversion coefficient A. Normally a black hole, with density of the order of 10^{18} kg/m^3, so that even light cannot escape from it, may be regarded as formed after numerous cycles.

It can be concluded that to attain such a state a quasar must undergo a long series of exceptionally intense compressions using the energy produced in itself. But the energy used for this purpose (internal changes) is not taken into account in current measurements of luminous energy, implying that the total energy (including measurable and immeasurable) is far higher than current estimates, i.e., A_{qu} may be more than 4×10^{16} (only luminous energy). This large amount of energy emitted by quasars and other heavenly bodies is consistent with $\Delta E = Ac^2\Delta M$, with higher values of A. Similarly, energy emitted by supernovas and other bodies can be explained. Thus, according to the equation $\Delta E = Ac^2\Delta M$, more energetic and abundant such explosions in the universe are feasible and the universe is more long-lived compared to predictions of $\Delta E = \Delta mc^2$, as for a smaller mass a huge amount of energy is emitted.

8.4.7 Dark Matter

There is compelling evidence that the predominant mass of the universe may be in the form of nonluminous "dark matter." The candidates for dark matter are numerous and include low-mass dwarf stars, neutron stars, hydrogen, black holes, massive neutrinos, magnetic monopoles, particles predicted by supersymmetry (the undetected gravitinos and photinos), and undetected axions (particles with extremely small masses). The above explanation (for annihilation of antimatter) of $\Delta E = Ac^2\Delta m$ favors dark matter in the form of the lightest neutrinos, axions, etc. Thus the bulk of the mass of antecedents according to $\Delta E = Ac^2\Delta m$ (value of A less than unity) is annihilated and small energy is emitted, so they reduce to the lightest descendants, e.g., neutrinos, axions, etc., which is consistent with existing perceptions. The reason for and effect of dark energy was already explained.

8.4.8 Black Holes

On the basis of $\Delta E = Ac^2\Delta m$, the reason for the formation of a black hole is that, due to the annihilation of mass, the temperature rises, and the energy thus produced is converted into gravitational energy. For example, in some huge heavenly bodies, a star with mass five solar masses is a prerequisite, as a considerable amount of mass is required to be annihilated for energy to be created (rise in temperature) and to be transformed to gravitational energy as in (62).

Thus the gravitational energy of the star became higher and higher, contracting its size and not even allowing light to escape. This process of annihilation of mass to energy then transformation of created energy to gravitational energy continues causing compression (increase in density) to an optimum limit. Thus, eventually, the black holes, whose gravitational energy tries to contract them beyond the optimum limit, may end in the form of a baby or black bang in the universe. According to this perception, even smaller heavenly bodies may become black holes if the value of A is sufficiently high and the value of k is small in (62). Thus for small-mass annihilation, a huge amount of gravitational energy is gained by the body, which does not even allow light to escape. The lightest black holes reside in the youngest galaxies.

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Résumé

La dérivation d'Einstein (septembre 1905) théorise que quand l'énergie de la lumière (L) est émanée par un corps lumineux alors sa masse diminue en accord avec $\Delta m = L/c^2$ et cette équation est d'origine spéculative (sans preuve) de $\Delta E = c^2 \Delta m$. La même dérivation prévoit que la masse d'un corps lumineux augmente en soi ($\Delta m = -0.034\ 90L/cv + L/c^2$) quand il émet l'énergie lumineuse; dans certains cas, la masse du corps demeure égale ($\Delta m = 0.$). Une équation alternative $\Delta E = Ac^2 \Delta M$ a été suggéré, ce qui implique que l'énergie émise après l'annihilation de la masse (ou vice versa) peut être soit égale à, moins que, ou plus que prévu par $\Delta E = c^2\ \Delta m$. On trouve, par les expériences, que l'énergie cinétique totale des fragments de fission de U^{235} ou de Pu^{239} paraisse être 20–60 MeV de moins que la valeur de Q telle que prévue par Δmc^2. Ceci pourrait être expliqué par l'équation $\Delta E = Ac^2\ \Delta m$ où la valeur de A étant moins que 1. $\Delta E = c^2 \Delta m$ n'a pas encore été confirmé dans des réactions chimiques. L'énergie émise par les éclatements de rayons gamma (événements les plus énergiques après le big bang) d'une durée de 0.1–100 s, est 10^{45} J, qui ne peut pas être expliqué par $\Delta E = \Delta mc^2$, comme il arrive dans les cas des quasars. Elle peut être expliquée par la valeur élevée de A, c.-à-d., 2.57×10^{18}. La masse de la particule Ds (2317) découverte au SLAC, est moindre que celle des estimations courantes. Ceci peut être expliqué par la valeur de A étant plus élevé que 1. L'équation $\Delta E = Ac^2\Delta M$ est utilisé pour expliquer que la masse de l'univers 10^{55} kg fut créée d'une énergie décroissante (10^{-444} J ou moins) et que A est de 2.568×10^{-471} J ou moins, et à la fin peut réduire vers une énergie petite. Ceci *explique le big bang, l'annihilation de l'antimatière dans l'époque du hadron, les trous noirs, la matière noir, etc. En ce qui concerne l'origine de l'énergie de la gravité inhérente, elle implique que c'est une autre forme de masse comme d'autres énergies, par conséquent la gravité et la masse sont inséparables.*

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