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The Various Equations of Variation of Mass with Velocity

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Abstract

Objectives: The origin of various equations involving variation of mass with velocity is discussed and new exponential equation is derived. At lower velocity this equation and Lorentz equation both give same results. **Methods/Statistical Analysis:** The various references right from inception of concept of variation of mass with velocity are discussed. The basic common point in various equations is that invalid operation division by zero is involved. Initially such equation was initiated by Thomson, and used by following scientists. Thus aspects are theoretically discussed. **Findings:** A newly derived equation is exponential in nature and is interpreted in view of existing experimental observations. It does not involve division by zero, hence never predicts that mass becomes infinite when velocity of body, v =c. Lorentz has given equation for transverse mass $m_T = \varepsilon \gamma m_{rest}$, where is undetermined factor or coefficient differing from unity by quantity of the order v^2/c^2. Lorentz's equation (relativistic mass) is experimentally verified by with reasonable accuracy up to velocity 0.75c. Thus Lorentz's equation is confirmed in limited region. In LHC the protons have energy 13TeV move with velocity at about 0.9999 99990c, at this velocity the relativistic mass of proton must be experimentally measured and compared. Then it must be confirmed up to which extent Lorentz's equation is obeyed. New theory of variability of speed of light implies that speed of light was more in the early universe. It supports exponential equation which allows superluminal velocity. **Applications/ Improvements:** The exponential equation is the first equation which provides extension in the Lorentz equation in conceptual and mathematical way. It stresses superluminal velocities at some stages of formation of universe. The exponential equation can be checked in experiments in LHC which involve velocities tending to that of light and other experiments.

Keywords: Einstein, Exponential Equation, Lorentz, Relativistic Mass, Transverse Mass

1. Introduction

The concept of variation of mass with velocity is gradually developed by many scientists. Initially it was concluded while studying phenomena in hydrodynamics that inertia of moving body in an incompressible perfect fluid increases[1]. Further it was recognized that a charged sphere moving in a space filled with a medium of a specific inductive capacity is harder to set in motion than an uncharged body[2]. This idea was worked out in more details[3–9] and was directly applied to the electron by using the Abraham–Lorentz force. Now it is also applied for neutral particles for various velocities. The perception of increase in mass with velocity had been increasingly refined by following many scientists.

(i) Thomson (1893) put forth that the momentum of the sphere and dielectric parallel to z is mw+I, where, m is the mass of the sphere; so that the effect of the charge will be to increase the apparent mass of the sphere by I/w or by[4].

$$\frac{1}{2}\frac{e^2}{a}\frac{V^2}{w\left(V^2-w^2\right)^{\frac{1}{2}}}\left[v\left(1-\frac{1}{2}\frac{V^2}{w^2}\right)+\frac{1}{2}\sin 2\theta(1+\frac{1}{4}\frac{V^2}{w^2}\cos 2\theta)\right] \tag{1}$$

In[4] '*When in the limit v = c (w=V), the increase in mass is infinite, thus a charged sphere moving with the velocity of light behaves as if its mass were infinite, its velocity therefore will remain constant, in other words it is impossible to increase the velocity of a charged body moving through the dielectric beyond that of light.*'

Thus the perception that nobody can move with velocity equal to that of light was originated by Thomson in an equation involving division by zero. And other

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scientists simply continued it. The division by zero is not permissible; this aspect is not taken in account by the originator, Thomson. This deduction is continuing ever since and scientists have given different equations for variation of mass without addressing the basic issue of invalidity of division by zero. Currently author has derived an exponential equation of variation of mass with velocity. At lower velocities 0.01c, both exponential and Lorentz equations give same results.

(ii) In[5] gave a more precise formula for the electromagnetic energy of charged sphere in motion as

$$E_{em}{}^{v} = E_{me}\left[\frac{1}{\beta}\ln\frac{1+\beta}{1-\beta} - 1\right]\beta = \frac{v}{c} \quad (2)$$

Like Thomson Searle also concluded that …. *nobody can move with speed equal to that of light, as denominator becomes zero.* It was concluded that the total mass of the bodies is identical to its electromagnetic mass[8].

(iii) From Searle's formula Walter Kaufmann and Abraham derived for the electromagnetic mass of moving bodies[5].

$$m_L = \frac{3}{4}m_{em}.\frac{1}{\beta^2}[-\frac{1}{\beta^2}\ln\frac{(1+\beta)}{(1-\beta)} + \frac{2}{1-\beta^2}] \quad (3)$$

(vi) Abraham confirmed that this value of mass is only valid in longitudinal direction (longitudinal mass) i.e. electromagnetic mass also depends upon direction of bodies with respect to another. Abraham derived the "transverse mass"[6].

$$m_T = \frac{3}{4}m_{em}\frac{1}{\beta^2}\left[\left(\frac{1+\beta^2}{2\beta}\right)\ln\left(\frac{1+\beta}{1-\beta}\right) - 1\right] \quad (4)$$

(v) On the other hand, already in 1899 Lorentz assumed that the electrons undergo length contraction in the line of motion, which leads to results for the acceleration of moving electrons that differ from those given by Abraham. Lorentz obtained factors of $k^3 \varepsilon$ parallel to direction of motion and $k \varepsilon$ perpendicular to direction of motion, where,

$k - = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ and ε is undetermined factor or coefficient of proportionality. In general, *ε*, is an undetermined factor (coefficient) *differing* from unity[7,8] by a quantity of the order v^2/c^2. Lorentz expressed in 1899, the longitudinal mass and transverse mass as in the following way,

$$m_T = \varepsilon\frac{m_{em}}{\sqrt{1-\frac{v^2}{c^2}}} \quad (5)$$

$$m_L = \varepsilon\frac{m_{em}}{\sqrt{(1-\frac{v^2}{c^2})^3}} \quad (6)$$

In basic physics the mass is scalar quantity, it never depends on direction. In relativistic physics mass depends upon velocity. However, Lorentz had put forth mass depends upon both direction and velocity. The transverse mass (perpendicular to the direction of motion) should have been more than longitudinal mass (parallel to the direction of motion). In derivation of rest mass energy $E_{rest} = M_{rest}c^2$, transverse mass is considered, not longitudinal mass which is less. Does it mean only transverse mass is converted to energy not longitudinal mass? The deductions should not be arbitrary. Or should we have transverse energy and longitudinal energy. Both the masses are equally probable and inseparable from each other. Further Lorentz set the factor ε to unity arbitrarily, thus[8]

$$m_T = \frac{m_{em}}{\sqrt{1-\frac{v^2}{c^2}}} \quad (7)$$

$$m_L = \frac{m_{em}}{\sqrt{(1-\frac{v^2}{c^2})^3}} \quad (8)$$

Further if velocity of body is 0.9c, then $m_T = 2.2941m$, and $m_L = 1.73947m$. Thus in magnitude transverse mass is more. Both masses are equally probable, thus must be used in calculations and interpretations equally. Similarly, in eqs. (7-8) the value of ε is used as unity, scientifically its exact value must be measured. The electromagnetic mass and mass (as we understand in general) are the same.

The value of ε is considered unity (in general ε is regarded as coefficient differing from unity by a quantity of order v^2/c^2). It is not clear whether peer review before publication of the paper was pre-requisite or not at that time. If exact values of ε are determined and substituted in eqs. (5-6), then values of m_T and m_L differ from current estimates.

(vi) Einstein wrote to Barnett[10]

"It is not good to introduce the concept of the $M_{rel} = M_{rel} = \frac{M_{rest}}{\sqrt{1-\frac{v^2}{c^2}}}$ mass of a moving body for which no clear definition can be given. It is better to introduce no other mass concept than the 'rest mass' M_{rest}. Instead of introducing *M* it is better to mention the expression for the momentum and energy of a body in motion[11]." It is strange suggestion by Einstein, as he had used the equation[12] in derivation of rest mass energy, $E_{rest} = M_{rest}\ c^2$. Further Einstein, like Lorentz, Abraham, Kaufman, Searle, Thomson etc. had given longitudinal and transverse masses, as given in eqs. (9-10). But Einstein did not apply the equations derived by him in any phenomena, he used Lorentz's equation i.e. eq. (7).

(vii) In the paper known as special theory of relativity[13] derived different equations of longitudinal mass and transverse mass as

$$\text{Longitudinal mass} = \frac{m}{\left[\sqrt{1-\frac{v^2}{c^2}}\right]^3} \tag{9}$$

$$\text{Transverse mass} = \frac{m}{1-\frac{v^2}{c^2}} \tag{10}$$

But in calculation of rest mass energy the transverse mass as given by Lorentz is quoted i.e. eq. (7), not eq. (9) or eq. (10).

(viii) Additionally, a third electron model was developed[8,14], in which the electron contracts in the line of motion, and expands perpendicular to it, so that the volume remains constant. This gives:

$$m_T = \frac{m_{em}}{\left(\sqrt{1-\frac{v^2}{c^2}}\right)^{\frac{2}{3}}} \tag{11}$$

$$m_L = \frac{m_{em}(1-\frac{1}{3}\frac{v^2}{c^2})}{\left(\sqrt{1-\frac{v^2}{c^2}}\right)^{\frac{8}{3}}} \tag{12}$$

Thus mass is relative i.e. it increases when body moves with speed equal to that of light. But different scientists have given different equations for relativistic mass. The various equations describing variation of mass with velocity are shown in Table 1.

Table 1. The various equations describing variation of mass with velocity

Sr. No	Scientist	Equation of variation of mass with velocity	Division by zero
1	J.J. Thomson, (1893)	$\frac{1}{2}\frac{e^2}{a}\frac{V^2}{w(V^2-w^2)^{\frac{1}{2}}}\left[v\left(1-\frac{1}{2}\frac{V^2}{w^2}\right)+\frac{1}{2}\sin 2\theta(1+\frac{1}{4}\frac{V^2}{w^2}\cos 2\theta)\right]$	Yes
2	G.F. Searle (1897)	$E_{em}{}^{v} = E_{em}\left[\frac{1}{\beta}\ln\frac{1+\beta}{1-\beta}-1\right]\beta = \frac{v}{c}$	Yes
3	M Abraham (1903)	$m_L = \frac{3}{4}m_{em}.\frac{1}{\beta^2}[-\frac{1}{\beta^2}\ln\frac{(1+\beta)}{(1-\beta)}+\frac{2}{1-\beta^2}]$ $m_T = \frac{3}{4}m_{em}\frac{1}{\beta^2}\left[\left(\frac{1+\beta^2}{2\beta}\right)\ln\left(\frac{1+\beta}{1-\beta}\right)-1\right]$	Yes
4	H A Lorentz (1899)	$m_T = \frac{m_{em}}{\sqrt{1-\frac{v^2}{c^2}}} m_L = \varepsilon\frac{m_{em}}{\left[3\sqrt{1-\frac{v^2}{c^2}}\right]}$	Yes
5	A Bucherer (1904)	$m_T = \frac{m_{em}}{\left(\sqrt{1-\frac{v^2}{c^2}}\right)^{\frac{2}{3}}} m_L = \frac{m_{em}(1-\frac{1}{3}\frac{v^2}{c^2})}{\left(\sqrt{1-\frac{v^2}{c^2}}\right)^{\frac{8}{3}}}$	
6	H A Lorentz (1904)	$m_T = \frac{m_{em}}{\sqrt{1-\frac{v^2}{c^2}}} m_L = \frac{m_{em}}{\left[3\sqrt{1-\frac{v^2}{c^2}}\right]}$	Yes
7	A Einstein (1905)	$m_T = \frac{m}{1-\frac{v^2}{c^2}} m_L = \frac{m}{\left[\sqrt{1-\frac{v^2}{c^2}}\right]^3}$	Yes
8	Sharma	$M = M_{rest}\, e^{\frac{Qv^2}{2c^2}}$	No

(vii) They recalled[15] "Friedman noticed that Einstein had made a mistake in his alleged proof that the universe must necessarily be static". Specifically, Einstein had divided an equation by a certain quantity, even though that quantity was zero under a certain set of conditions. As Gamow notes, *"it is well known to students of high school algebra that division by zero is not valid"*. It should have been noticed by Thomson in the onset and then by following scientists. Years later Einstein recalled the sequence of events[15], and made the famous remark that it had been the biggest blunder of his life. The comparison of given by Einstein and Lorentz is given in the Table 2.

2. Theoretical and Experimental applications of Eq. (7)

(i) Successively more and more refined experiments have been conducted to confirm eq. (7), neglecting similar other equations. In this regard the most reliable experiment had been reported[16]. In the paper just three points are reported for velocities 0.5c, 0.7c and 0.75c (maximum speed of electron) and concluded that it is established within accuracy of 1%. After Roger's experiments no specific experiments have been reported in this regard (variation of mass with velocity) involving velocities tending to speed of light[17]. Moreover, to confirm such an equation large number of observations is needed with velocities tending to c. But no such detailed information is available[17]. These observations at highest velocity 0.75c may not be regarded as scientifically complete or adequate. Also in particle accelerators and other experiments, the predictions of special relativity (length contraction and time dilation) are regarded as confirmed . These issues are not debated. However we are specifically discussing the precise experimental confirmation of equation of relativistic variation of mass (ε =1) i.e. eq.(7). An equation is said to be experimentally established, if justified for all possible parameters. Thus specific experiments for measuring mass of particles moving at various velocities i.e. 0.1c, 0.4c, 0.8c, 0.9c, 0.9999c etc. more are required. If the variations of 1% is found (as in case of Roger's experiment) at velocity approaching to that of light, then it would be interesting to interpret. Should we try to explain such results with help of eq.(5) which contains additional indeterminate coefficient ε.

(ii) The protons are accelerated to energy 6.5 TeV in Large Hadron Collider; the protons each have an energy of 6.5 TeV, giving a total collision energy of 13 TeV. At this energy the protons have a Lorentz factor of about 7071 and move at about 0.9999 99990 c, or about 3.1 m/s (11 km/h) slower than the speed of light[18].

This velocity of protons was not measured experimentally but theoretically from by solving equation of relativistic kinetic energy[19],

$$KE = (M_{rel} - M_{rest})c^2 \tag{13}$$

$$\text{or } v = c\sqrt{1-\left[\frac{1}{1+\dfrac{KE}{m_{rest}c^2}}\right]^2} \tag{14}$$

This equation needs to be critically checked .This eq.(14) is based on the fact that nobody can move with

Table 2. Transverse and Longitudinal masses given by Lorentz and Einstein

Characteristics	Transverse mass	Longitudinal mass
Einstein's m_T and m_L	$m_T = \dfrac{m}{1-\dfrac{v^2}{c^2}}$	$m_L = \dfrac{m}{\sqrt{(1-\dfrac{v^2}{c^2})^3}}$
Applications	$KE = Mc^2 - Mv^2/2$	none
Lorentz's m_T and m_L	$m_T = \varepsilon\dfrac{m_{rest}}{\sqrt{1-\dfrac{v^2}{c^2}}}$	$m_L = \varepsilon\dfrac{m_{rest}}{\left[\sqrt{1-\dfrac{v^2}{c^2}}\right]^3}$
ε=1	$m_T = \dfrac{m_{em}}{\sqrt{1-\dfrac{v^2}{c^2}}}$	$m_L = \dfrac{m_{rest}}{\left[\sqrt{1-\dfrac{v^2}{c^2}}\right]^3}$
Applications	$E = Mc^2$ if ε=1	Not applied yet

Note : In general[7] ε is regarded as coefficient differing from unity by a quantity of order v^2/c^2

speed of light i.e. eq. (7), and predicts that even if KE = ∞, then body moves with speed of light.

Thus it is suggested that the velocity of protons must be measured by direct method not theoretically by eq. (14), which predicts that velocity of body always remain less than c. When contacted that should we measure the velocity of protons by direct experimental method, the answer of CERN Control Centre (CCC) was affirmative. CERN stated that[20],

In short : measuring the revolution frequency , knowing path length , we can derive the actual velocity.

$$v = rw = r\, 2\pi f \qquad (15)$$

If the neutrinos (rest mass tending to zero) are accelerated with such high energy (even more) and velocity is measured directly then it may theoretically exceed c. Thus theoretically eq. (15) predicts that the speed of particle can be more than c, whereas according to eq. (14) speed of particle has to be less than c. This is the conceptual difference in predictions of eq. (14) and eq. (15).

(iii) So the eqs. (7,8) are based upon assumption ε =1, but value of ε is not determined experimentally and theoretically. If various values of ε are taken then magnitude of eq.(5) also varies. In the existing physics value of ε other than unity is not considered. For example in derivation of rest mass energy $E_{rest} = M_{rest}\, c^2$ the value of ε is assumed to be unity by Einstein in eq. (5). Also eq. (7) is experimentally justified where value of ε is regarded is unity. Originally Lorentz had regarded ε as coefficient differing from unity by a quantity of order v^2/c^2.

3. Mathematical Results based on Einstein's Transverse Mass Equation i.e. eq.(10).

Only Lorentz's equation of transverse mass i.e. eq.(7) is used as relativistic mass in all calculations. Einstein's equation of transverse mass i.e. eq. (10) but it differs in magnitude from Lorentz's equation i.e. eq. (7). Einstein did not use equation derived by him i.e. eq. (10) but applied Lorentz's equation. If velocity of body is regarded 0.5c then

$$m_T \text{ (Lorentz)} = 1.1547\, M_{rest} \qquad (16)$$
$$m_T \text{ (Einstein)} = 1.3333\, M_{rest} \qquad (17)$$

If other equations are employed in calculations then values of transverse masses would be different. The same equation can also be used in related mathematical calculations and results can be analyzed. The relativistic energy (E energy, M relativistic mass, m rest mass) is determined by using transverse mass i.e. eq. (7) as

$$dK = dW = Fdx = \frac{d(Mv)}{dt} = (Mvdv + v^2dM) \qquad (18)$$

Now squaring and differentiating eq. (7) we get

$$c^2dM = (Mvdv + v^2dM) \qquad (19)$$

$$\text{or } dK = dW = c^2dM \qquad (20)$$

Comparing eq. (18) and eq. (20)

$$K + mc^2 = Mc^2 \text{ or } E = Mc^2 \qquad (21)$$

$$\text{or KE} = \frac{mv^2}{2} \qquad (22)$$

The Relativistic energy from Einstein's transverse mass i.e. eq. (10). In identical way squaring and

Differentiating eq. (10) as

$$c^2dM = (2Mvdv + v^2dM) = (v^2dM + Mvdv + Mvdv)$$
$$c^2dM - Mvdv = (v^2dM + Mvdv) \qquad (23)$$

Comparing eq. (18) with eq. (23)

$$\text{or } dK = dW = c^2dM - Mvdv$$

$$K = c^2(M - m) - \frac{Mv^2}{2}$$

$$\text{or } K + mc^2 = Mc^2 - \frac{Mv^2}{2} \qquad (24)$$

Substituting eq. (10) in eq. (24) and applying Binomial Theorem under classical conditions (v<<c) and neglecting higher terms we get

$$KE = mc^2 + mv^2 - mc^2 - mv^2/2 + \ldots\ldots\ldots\ldots = \frac{mv^2}{2} \qquad (22)$$

which is classical form of kinetic energy. Thus under classical conditions (v<<c) Lorentz's equation and Einstein's equation give same results. Similarly, the relativistic form of kinetic energy in this case can be written as

$$E = Mc^2 - \frac{Mv^2}{2} \quad (25)$$

Both types of equations i.e. based on eq.(7) and eq.(10) are equally feasible mathematically.

4. Some Typical Results

(i) Some peculiar results can be deduced from eq. (7). According to eq. (7) a body of mass, even $10^{-999999999}$ kg or less (immeasurably small) can never attain speed equal to that of light, as mass becomes infinite instantaneously. The infinite mass will be accommodated in the infinite space; thus no other particle can attain speed equal to that of light. Does infinite space exist or will be formed when velocity would attain v=c? When v=c then a particle of mass $10^{-999999999}$ kg become infinite. Obviously, its dimensions (length, breadth and height) will be infinite as well, as a body of finite dimensions cannot have infinite mass.

(ii) But under that condition according to length contraction length ($L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$) of particle becomes zero. How a body of infinite mass can have zero length (mass = ∞ and length = 0)? Ideally, it can be concluded that if length of body is zero, then its mass must be zero. Thus an unphysical result (i.e. length of body is zero and mass is infinite) is obtained when both deductions of special theory of relativity are simultaneously compared. Lorentz reasoned that mass of charged particle would increase as its length decreases. He assumed that mass of charged particle was due to potential energy of its own charge[21]. Even primeval atom (infinite density and temperature) has atomic dimensions and mass is regarded as 10^{55} kg (estimates may vary).

(iii) A photon is an elementary particle, the quantum of all forms of electromagnetic radiation including light. The photon has zero rest mass and as a result, the interactions of this force with matter at long distance are observable at the microscopic and macroscopic levels. Like all elementary particles, photons are currently best explained by quantum mechanics and exhibit wave–particle duality, exhibiting properties of both waves and particles. The relativistic mass of photon is less than 10^{-18} eV/c^2 (10^{-54} kg) and electronic charge less than 10^{-35}e (1.602 x10^{-53} C).

(vi) The speed of light (source of light, Sun) had different values at different times Romer and Huygen (1675) measured speed of light as (2.2 x10^8m/s), Rosa Dorsey (1907) as ((2.299710x10^8 km/s) and the current value of speed of light adapted in 17th CGPM (1983) (2.299792x10^8km/s). The speed of light must be measured from various sources of light e.g. firefly (biological source), candle (chemical source), laser (technological sources), moon (reflected light), light emitted from the violent supernova explosions or any other source of light. Initially Poincare and preceding scientists[22] meant the constancy of speed of light for heavenly bodies i.e. stars etc. Whereas Einstein used it in general sense i.e. any ray of light moves with speed equal to c, which is true for all sources of light.

According to second reflexion or postulate of Einstein's theory of relativity[13],

Any ray of light moves in the "stationary" system of co-ordinates with the determined velocity c, whether the ray be emitted by a stationary or by a moving body. Hence

$$\text{Velocity} = \text{light path} / \text{time interval} \quad (26)$$

The postulate is same for all sources of light. Thus it must be measured for 'every possible' sources of light and results must be same. If value of c differs for various sources, then it would be interesting to interpret results. A new theory of variable speed of light has been proposed, this model experimentally could be put to test[23]. This perception implies that speed of light was more at time of big bang and decreasing now.

5. A New Equation for Relativistic or Non Relativistic Variation of Mass: $M = M_0\, e^{\frac{Qv^2}{2c^2}}$

In relativistic physics Thomson (1893), Heaviside, Searle, Kauffman, Abraham, Lorentz (1904), Bucherer, Einstein (1905) etc. have given different equations of variations of mass with velocity. All the equations involve an invalid mathematical operation i.e. 'division by zero'. Thomson was the first scientist who derived such equation.

Science would be static without speculation. Can we speculate a sophisticated experimental set up where a photon can be accelerated to speed more than that of light? The experimental techniques may not have been enough energetic to provide adequate energy to the lighter particles to move with speed of light. Similarly, an equation is derived which describes variation of mass with velocity but division by zero does not occur. The newly derived equation is exponential in nature predicts mass increases with velocity exponentially.

The exponential equation of variation of mass with velocity is given by

$$M = M_{rest} e^{\frac{Qv^2}{2c^2}} \quad (30)$$

where, M is mass of body moving with velocity v (M_{rel}), M_{rest} is rest mass, c is speed of light and Q is co-efficient used while removing proportionality.

6. Derivation

We can assess that the variation in mass with velocity is proportional to the velocity v of body and mass M. Thus, we can write variation in mass with velocity v as

$$\frac{dM}{dv} \alpha\, M \quad \frac{dM}{dv} \alpha\, v \quad or \quad \frac{dM}{dv} \alpha\, Mv \quad or \quad \frac{dM}{M} = Kvdv \quad (27)$$

where, K is coefficient of proportionality and depends upon inherent experimental conditions of the process. It has dependence like resistance (V=IR or R = ρL /a, ρ resistivity, L length and a is area of conductor) in the existing in the literature. Similarly, in Hubble Law i.e. V=HS, the Hubble's coefficient have different values in different measurements. Thus age of universe (inverse of Hubble's constant) is not fixed, it is variable like Hubble's coefficient or parameter. There are many coefficients of proportionality i.e. coefficient of viscosity, coefficient of thermal conductivity, coefficient of elasticity, coefficient of expansion etc. etc. The value of K may be assessed as

$$K\ \alpha\ 1/c^2 \text{ or } K = Q/c^2 \quad (28)$$

The value of K is non-zero. Now integrating within limits (mass varies from M_{rest} to M or M_{rel}, and velocity 0 to v).

$$\int_{M_{rest}}^{M} \frac{dM}{M} = \frac{Q}{c^2}\int_0^v vdv \quad (29)$$

$$\ln M - \ln Mrest = Qv^2/ 2c^2$$

$$\text{or } M=M_{rest}\, e^{\frac{Qv^2}{2c^2}} \quad (30)$$

7. Theoretical Discussion of Photons in View of eq. (7) and eq. (30)

The photon is regarded as mass less i.e. $M_{rest} = 0$ and let it be moving with speed equal to that of light. Theoretically, according to Newton's second law F=ma, a particle having rest mass zero can move with exceedingly high velocity then eq. (7) becomes

$$M = \frac{0}{0} \quad (31)$$

which is indeterminate form. However, mass of photon (when moving) is 10^{-18} eV/c^2 (10^{-54} kg) as estimated and charge 10^{-34} e. If the rest mass of photon is zero, then how does it attain non-zero mass (10^{-54} kg) and charge (10^{-34} e)? Now an undefined mass cannot be 10^{-54} kg and having charge equal to 10^{-34} e. If this mass is converted from energy ($E=mc^2$) then it should arise from materialization of energy $9x10^{-38}$ J. Next question is how this energy is produced? Now origin of this energy and mode of conversion of energy to mass and charge has to be properly resolved. If the mass equal to 10^{-54} kg move with speed equal to that of light then its relativistic mass becomes ∞. To understand this in one of the way we can speculate that rest mass of photon be $10^{-9999999999999}$ kg or less i.e. tending to zero (immeasurably small). But under this condition eq. (7) gives infinite relativistic mass.

These inconstant results are due to fact the denominator of eq. (7) becomes zero i.e. division by zero occurs. However it is not so in case of equation involving exponential variation of mass as

$$M = M_{rest}\, e^{\frac{Qv^2}{2c^2}} = 10^{-999999999999}\, e^{\frac{Q}{2}} \text{ kg} \quad (32)$$

If the value of Q is regarded as unity, then

$$M = 1.648x10^{-999999999} \text{ kg} \quad (33)$$

Hence, mass increases with velocity and results are logical. It implies that when body moves its mass increases, then it may be perceived in general way that

'motion and formation of universe are two simultaneous events.'

8. Comparisons and Deductions

Both equations give variation of mass with velocity. In eq. (7) the division by zero occurs but not in eq. (30) it does not. At lower velocities, both equations give same results. Strictly speaking, Lorentz's equation i.e. eq. (7) is not experimentally confirmed for all possible parameters. Realistically actual value of ε should be given adequate consideration. At higher velocities the results of eq. (30) also coincide with eq. (7) if values of Q are empirically determined.

(i) If v = 0, then both the equations predict

$$M = M_{rest} \tag{34}$$

which is consistent.

(ii) At lower velocities: Further both the equations (when Binomial Theorem is applicable) relating variation of mass with velocity can be written as

$$M = \frac{M_{rest}}{\sqrt{1-\frac{v^2}{c^2}}} = M_{rest}\left[1+\frac{1}{2}\frac{v^2}{c^2}+\frac{3}{8}\frac{v^4}{c^4}+.......\right] \tag{35}$$

$$M_{rest}e^{\frac{v^2}{2c^2}} = M_{rest}\left[1+\frac{1}{2}\frac{v^2}{c^2}+\frac{1}{8}\frac{v^4}{c^4}+.......\right] \tag{36}$$

(iii) **If velocity, v =0.04c:** The eq.(7) i.e. eq.(5) with value of ε as unity and eq. (30) with value of Q as unity, lead to similar results at velocities up to nearly 0.04c ($1.2\text{x}10^7$ m/s) i.e. comparable to that of light. If the value of ε in eq.(5) and Q in eq.(30) is regarded as unity , then we have

From eq.(30) $M = 1.00080\ M_{rest}$ (37)

From eq.(7) $M = 1.00080\ M_{rest}$ (38)

The value of ε in eq.(38) is unity.

Thus both equations give the same results. Then both equations coincide up to velocities 0.04c or $1.2\text{x}10^7$m/s or $4.32\text{x}10^7$ km/hr. Even in experimental verification of transverse mass as given by Lorentz i.e. eq. (5), the value of ε is regarded as unity. Thus eq. (7) is justified with accuracy of 1%. So all tests have experimental constraints.

If velocity, v=0.05c: Under this condition eq. (30) and eq. (5) become

From eq.(30) $M = 1.00125M_{rest}$ (39)

From eq.(7) $M = 1.00125M_{rest}$ (40)

(vi) **If velocity , v=0.1c :** Under this condition eq.(30) and eq.(5) become

From eq.(30) $M = 1.00501\ M_{rest}$ (41)

From eq.(7) $M = 1.00503M_{rest}$ (42)

Thus difference between two equations is $2\text{x}10^{-3}$. In eq. (30) the value of coefficient Q is assumed to be unity. Both the equations will give same results if value of Q is 1.00503.

9. Empirical Determination of Coefficient of Proportionality, Q

The magnitudes of mass (if v=0.2c) from eq. (7) and eq. (30) are 1.0206207 M_{rest} and 1.0202013 M_{rest} respectively. Thus % age difference between both values $4.19\text{x}10^{-2}$. Both the equations would give similar results if Q =1.02054.The value of ε is assumed to be unity in eq.(5) in all calculations.

$$M_{rest}e^{\frac{Qv^2}{2c^2}} = \frac{M_{rest}}{\sqrt{1-\frac{v^2}{c^2}}} \quad or \quad Q = \frac{2c^2}{v^2}1n\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{43}$$

Q= 1.02054 if v=0.2c

At the higher velocities the eq. (30) predicts that mass is consistent with eq. (7) if the value of coefficient Q is empirically determined. As it is firmly believed by scientists that nobody can move with speed of light, so under this condition the value of Q is exceptionally -2 high. The eq. (30) never predicts inconsistent results i.e. undefined, infinite or imaginary mass. Let us perceive that at some stage superluminal velocities are observed may be in laboratories or heavenly phenomena; then value of Q can be determined from actual experimental.

In the existing physics or science, the values of coefficients of proportionality are empirically determined e.g. the total binding energy E_b of nucleus is ought to be sum of volume, surface, coulomb energy, asymmetry and pairing energies. The semi empirical binding-energy formula obtained by C F Von Weizsacker is[24]

$$E_b = a_1A\text{-}a_2A^{2/3}\text{-}a_3\frac{(A-2Z)^2}{A}(\pm,0)\frac{a_5}{A^{\frac{3}{4}}} \tag{44}$$

The coefficients are empirically determined and various sets are proposed. The set of coefficients that gives a good fit with the data as follows:

a_1 =14MeV, a_2 =13MeV, a_3 =0.60MeV, a_4 = 19MeV, a_5 =34MeV

Similarly, the value of coefficient of proportionality Q is determined so that eq. (7) and eq. (30) are simultaneously obeyed. Due to value of the coefficient Q, Lorentz's equation i.e. eq. (7) and exponential equation i.e. eq. (30) give similar results. It should be noted that in eq.(7) value of ε is assumed to be unity. The eq. (7) has to be confirmed for all possible values of parameters to draw final conclusions. The values of Q are shown in Table 3.

Table 3. Equality of eq. (7) and eq. (30) , the value of Q is shown in last column

Sr No	Velocity	$M = \frac{M_{rest}}{\sqrt{1-\frac{v^2}{c^2}}} = M_{rest}e^{\frac{Qv2}{2c2}}$	The value of Q as in eq.(43)
1	v=0	M_{rest}	1
2	v=0.0001c	1.000000005 M_{rest}	1
3	v=0.001c	1.0000005M_{rest}	1
4	v=0.01c	1.00005 M_{rest}	1
5	v=0.04c	1.00080 M_{rest}	1
6	v=0.1c	1.00503 M_{rest}	1.00503
7	v=0.2c	1.02062 M_{rest}	1.020549
8	v=0.5c	1.15470 M_{rest}	1.150728
9	v=0.9c	2.29415 M_{rest}	2.05028
10	v=0.999991c	235.70226 M_{rest}	10.92515
11	v=0.99999999c	7071.06781 M_{rest}	17.72753
12	v $\rightarrow$c	M$\rightarrow\infty$	Unusually high

10. Conclusions

It is confirmed experimentally that no particle can move with speed of light. It may be due to reason that the light photons are the lightest of all. First of all, mathematically it was deduced by Thomson that if body attains speed of light, then its mass becomes infinite. Hence, nobody can move with speed of light, c or more than c. The following scientists including Lorentz gave mathematical equations for variation of mass with velocity, which also restrict any particle moving with speed equal to that of light. However, magnitudes of transverse and longitudinal masses are different in different equations. These equations involve invalid operation 'division by zero.' At the same time particles with superluminal velocities are perceived and discussed by scientists. Here an equation, M=M_{rest} $e^{\frac{Qv^2}{2c^2}}$ is derived for variation of mass with velocity such that mass does not become infinity and 'division by zero' does not occur when v=c. Also at lower velocities this equation gives same result as Lorentz equation, at higher velocity also if the value of Q is determined empirically. At CERN at LHC the high energy protons have velocities about 0.9999 99990c, thus mass of protons can be calculated and compared with both equations. In future higher velocities are expected thus experimental verification of equations may lead to interesting results. Further this equation can be applied in early cosmology.

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