Newton, Euler and the Second Law of Motion $F = ma$

Ajay Sharma*

The prevalent form of Newton's second law of motion involves acceleration as $F = ma$. But Newton's second law is given in Book I of the *Principia* at page 19-20 as the alteration in motion is proportional to force, *i.e.* $F = (v - u)$. But conceptually at Newton's time neither acceleration nor the second derivative were derived. So it was impossible for Newton to write $F = ma$; this equation was clearly stated by Euler in 1775, 48 years after the death of Newton. In the existing literature "quantity of motion" (amount or quantum or magnitude of motion) is regarded as "absolute motion" or motion, and "alteration" is regarded as "rate of change of..." Consequently *Principia's* second law of motion is regarded as $F = ma$, which is not consistent. In Book III of the *Principia* at pages 213-226 in Propositions I–VIII, the law of gravitation was described. Like the second and third laws of motion, in this case also no mathematical equation is given ($F = GmM/r^2$) for the law of gravitation. In fact that was the beginning of physics—elementary terms such as mass, space, absolute and relative motion, quantity of motion, inertia, force, etc. were conceptually defined. The laws were expressed in terms of elaborative explanation and illustrations. Newton explained his three laws of motion like this without equation. The analytical and algebraic equations were not written, which gives quantitative variation of one parameter when another changes. The equation of force currently used, *i.e.* $F = ma$, was given by Euler in 1775, and is completely independent of Newton's law. Euler's equation $F = ma$, regarded as the second law of motion after the death of Newton, is contradictory to concepts laid down by Newton in the *Principia*.

1. Axioms or Laws of Motion by Isaac Newton

Historically Aristotle (384-322 BC) asserted (nearly 2000 years before Newton) that speed is proportional to motive force, and inversely proportional to resistance.[1] Then the doctrine of Galileo dominated as formulated for resistanceless systems and the stage was set for Newton's laws. The *Principia* was initially written in the Latin in three editions (1687, 1713, 1726) and it was translated to English by Andrew Mott in 1729 two years after the death of Newton.[2] In the beginning Newton had conceptually defined various terms such as mass, space, absolute and relative motion, quantity of motion, inertia, force, etc. Thus was the beginning of science. Newton's laws along with explanation as given in the *Principia* are given below.[2] The acceleration and second derivative were not discovered in Newton's time.[3-4]

LAW I

Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.

Explanation to First Law of Motion by Newton in the Principia

Projectiles persevere in their motions, so far as they are not retarded by the resistance of the air, or impelled downwards by the force of gravity. A top, whose parts by their cohesion are perpetually drawn aside from rectilinear motions, does not cease its rotation, otherwise than as it is retarded by the air. The greater bodies of the planets and comets, meeting with less resistance in more free spaces, preserve their motions both progressive and circular for a much longer time.

LAW II

The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

Explanation to Second Law of Motion by Newton in the Principia

If any force generates a motion, a double force will generate double the motion, a triple force triple the motion, whether that force be impressed altogether and at once, or gradually and successively. And this motion (being always directed the same way with the generating force), if the body moved before, is added to or subtracted from the former motion, according as they directly conspire with or are directly contrary to each other; or obliquely joined, when they are oblique, so as to produce a new motion compounded from the determination of both.

LAW III

To every action there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

Explanation to Third Law of Motion by Newton in the Principia

Whatever draws or presses another is as much drawn or pressed by that other. If you press a stone with your finger, the finger is also pressed by the stone. If a horse draws a stone tied to a rope, the horse (if I may so say) will be equally drawn back towards the stone, for the distended rope, by the same endeavor to relax or unbend itself, will draw the horse as much towards the stone as it does the stone towards the horse, and will obstruct the progress of the one as much as it advances that of the other.

If a body impinges upon another, and by its force change the motion of the other, that body also (because of the quality of, the mutual pressure) will undergo an equal change, in its own motion, towards the contrary part. The changes made by these actions are equal, not in the velocities but in the motions of bodies; that is to say, if the bodies are not hindered by any other impediments. For, because the motions are equally changed, the changes of the velocities made towards contrary parts are reciprocally proportional to the bodies. This law takes place also in attractions, as will be proved in the next scholium.

Newton ended with only this explanation to his three laws of motion. Now it is clear that Newton has simply given statement and explanation (mostly qualitative) and no mathematical equation for the first, second and third laws, and the law of gravitation.

2. Useful Terms in the *Principia* to Understand the Second Law of Motion

(i) *Definition of Rest:* When the position of a body with respect to its surroundings does not change with time, it is said to be at rest. In this case, velocity $v = 0$ as distance moved is zero or body is motionless.

Definition of Motion: Motion is a change in position of an object with respect to time and its reference point. Thus in this case body moves $v > 0$ as distance moved is non-zero. When body is in motion it possesses velocity. So both the states of rest and motion are described in terms of velocity.

(ii) *Motion as Celerity (Velocity):* While explaining Def. VIII at page 8, Newton stated, "Wherefore the Accelerative force will stand in the same relation to Motive force, as celerity does motion." Thus motion is directly related to celerity (velocity or speed).

Acceleration: The term acceleration (the rate of change of velocity) was never discussed by Newton in the *Principia*.

(iii) *Absolute Motion (motus absolutus):* Newton[2] defined motion as absolute motion in Scholium of Def. VIII, page 10.

It is the translation of a body from one absolute space to another. In physics translation (uniform movement) it is defined as movement of body from one point to another, *i.e.* body possesses velocity.

Relative motion is the translation from one relative place to another.

Further in the same section is an understanding of the motion of a ship with respect to earth; Newton expressed it in terms of velocity.

Absolute motion = motion velocity or speed

$$\text{Thus, alteration in motion} = v - u \qquad (1)$$

The absolute motion and relative motion are expressed in terms of velocity. Now these terms are called velocity or relative velocity.

(iv) *Quantis motus or Quantity (amount or quantum or magnitude) of motion:* "Quantity of motion' is defined in Def. II at page 2. Absolute motion or motion is defined in Scholium after Def. VIII at page 10 at para IV. Both must be clearly understood before interpretation. "The Quantity of Motion is the measure of the same, arising from the velocity and quantity of matter conjunctively."

Thus quantity (amount or quantum or magnitude) of motion is product of mass and velocity, also explained in the *Principia* just after definition. The quantity of motion means the amount or magnitude of motion, hence it is different from motion (translation of body from one space to another).

Both the terms, *i.e.* absolute motion (translation of body from one space to another) and quantity of motion (product of mass and velocity of body), are different, hence defined separately at different places in the *Principia*. Further relative motion is the translation (movement) from one relative place to other. The motion is not defined as product of mass and velocity neither in the *Principia* nor elsewhere by Newton. If the meaning of both is the same, then they would have been explained in the same line or sentence and should have units and dimensions. Thus the definition of quantity of motion and absolute motion are entirely different, hence defined under different headings. Thus "motion" cannot replace "quantity (amount or magnitude) of motion" and vice versa. These terms are not synonymous. It is also clear from the dictionary meaning of the words. Motion is described in terms of "velocity" and "quantity of motion" as product of mass and velocity. If both are regarded as having the same meaning, then it is misinterpretation and contradictory to concepts laid down by Newton in the *Principia*.

(v) A*lteration and "Rate of Change":* In the original definition of Newton's second law of motion as given in all three editions of the *Principia* (1687, 1712, 1726), the word "alteration" is used. It means change in two stages or states ($v_2 - v_1$). However, usually the "rate of change of" is used, instead of alteration, it is not justified. Both alteration (difference) and "rate of change" (variation with respect to time), *i.e.* first derivative, are entirely different, not similar. Thus Δv and $\frac{dv}{dt}$ are conceptually and mathematically entirely different. These terms cannot be regarded as replacements of each other. Even Newton did not write "rate of change of" in his work. So it is not logical to replace "rate of change of" with "alteration," as these are also not synonymous.

(vi) *Definition (IV) of Force:* "An impressed force is an action exerted upon a body, in order to change its state, either of rest or of moving uniformly forward in right line." Now, the state of moving uniformly forward in right line is state of uniform velocity forward in a right line.

Thus impressed force is related to velocity. This definition of the impressed force in other words is the first law of motion.[2] Also the impressed force is associated with the second law of motion. Thus in Newton's *Principia*, impressed force is associated with velocity. Now centripetal force is given by $mv^2/2$, which depends upon velocity v. It can be easily justified that in the first law of motion, the force is dependent on motion (velocity).

Raman stated: "By 'motion' Newton meant 'quantity of motion' which he had defined as the product of mass and velocity *i.e.* what he would call momentum. The crucial expression is 'change in momentum.' The usual tendency is to take this to mean 'rate of change of momentum.'"[5]

Newton defined "quantity of motion" and "motion" at different places, hence both are different, not synonymous. The dictionary meaning of quantity is "amount or quantum or magnitude." It cannot be ignored. Newton never meant "motion" as "quantity of motion" (momentum), and never wrote the equation $F = ma$ for the *Principia's* second law of motion. This aspect (whether motion and quantity of motion are the same) was discussed in the *Stanford Encyclopedia of Philosophy* as:

> The obvious question with the second law is what Newton means by "a change in motion." If he had meant a change in what we call momentum—that is, if he had meant, in modern notation, Δmv—the proper phrasing would have been "a change in the quantity of motion."[6]

It is correct that "motion" cannot be regarded as "quantity of motion," as both terms are conceptually different. The dictionary meaning of quantity of motion is "amount or extent or quantum or magnitude of motion." Newton defined it as product of mass and velocity. It is different from motion. Newton defined "quantity of motion" in Def. II at page 2 whereas "absolute motion" is defined in Scholium section IV after Def. VIII at page 10. Newton regarded both terms as different and hence defined at different stages. Newton defined the terms "absolute motion" and "relative motion" which implies absolute velocity or relative velocity. Thus "quantity of motion" and "motion" are entirely different terms and are not synonymous. It is amply clear from the *Principia* that motion (absolute motion, *i.e.* velocity) is entirely different from "quantity of motion," *i.e.* momentum.

Further the meanings of words or terms are not defined or expressed by intuitive or tendentious way; as stated by Raman,[5] these are defined on the basis of established concepts. There is further inconsistent deduction that change in velocity, Δv (*Principia's* second law) is taken as "rate of change of momentum" $\frac{dp}{dt}$ as implied from Raman's deduction.[5] In the *Principia* Newton did not write $\frac{dp}{dt}$ or $m\frac{d^2x}{dt^2}$. The change in momentum has units and dimensions (m/s and MLT^{-1}) and that of 'rate of change of momentum' (m/s^2 and MLT^{-2}). Thus $F = ma$ does not follow from the second law of motion as given in the *Principia*, and is an independent conclusion.[6] The scientific interpretations vary as the newer facts or results are revealed.

3. In the *Principia* *F* = *ma* Was Not Derived

In the *Principia* Newton did not give any mathematical equation for other established laws as well, *e.g.* for the third law of motion and law of gravitation. The equation for the second law of motion from the *Principia* follows, $F \alpha (v - u)$ or

$$F = (v - u) \tag{2}$$

In Newton's time it was not mandatory to express the laws in terms of mathematical equations, so he did not write equations for definitions in the *Principia*. Had Newton given equations in the *Principia* then the issue would have been solved easily. But later on the equations were arbitrarily written in the case of the second law of motion, in view of $F = ma$ (as derived by Euler). Also the concept of units and dimensions was developed centuries after the perception of the law. In Newton's time neither acceleration (rate of change of velocity) nor second derivative were defined, so it was impossible for him to write $F = ma$.

Now the prevalent form of second law of motion is "The rate of change of momentum is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed." This definition is entirely different from the P*rincipia's* second law of motion. In fact it is a text form of the equation $F = ma$ which was given by Euler. The *alteration in motion* $(v - u)$ cannot be *rate of change of momentum* $(\frac{dp}{dt})$ neither conceptually nor mathematically. Also both have different units and dimensions.

Newton started the beginning of the laying of the foundations of physics by defining the basic terms, such as mass, inertia, force, rest, motion, gravity, centripetal force many forms Def. V-VIII, etc. in the *Principia*. The term acceleration is not mentioned at all. At that time physical phenomena were expressed in terms of laws, axioms, propositions etc. not by algebraic or analytical equations. Newton wrote in Propositions (I-VIII) in Book III of the *Principia*[2] about force of attraction between various heavenly bodies. These are regarded as laws of gravitation. But Newton did not give any mathematical equation to the law of gravitation ($F = \frac{GmM}{r^2}$), the third law of motion; likewise an equation was not given for the *Principia's* second law of motion. In all three laws Newton gave explanation in text form not in mathematical form in the *Principia*. Thus it is a correct conclusion that Newton did not derive $F = ma$.

The formulation of equations and mathematical interpretations were the next phase of development of concepts, followed by experimental confirmations. It is concluded independently that Newton did not give $F = ma$ in the *Principia*. "The obvious question with the second law is what Newton means by 'a change in motion.' If he had meant a change in what we call momentum—that is, if he had meant, in modern notation, Δmv—the proper phrasing would have been 'a change in the quantity of motion.'"[6]

According to the *Stanford Encyclopedia of Philosophy*:

> The modern $F = ma$ form of Newton's second law nowhere occurs in any edition of the *Principia* even though he had seen his second law formulated in this way in print during the interval between the second and third editions in Jacob Hermann's *Phoronomia* of 1716. Instead, it has the following formulation in all three editions: A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed.
>
> If this way of interpreting the second law seems perverse, keep in mind that the geometric mathematics Newton used in the *Principia*—and others were using before him—had no way of representing acceleration as a quantity in its own right. Newton, of course, could have conceptualized acceleration as the second derivative of distance with respect to time within the framework of the symbolic calculus. This indeed is the form in which Jacob Hermann presented the second law in his *Phoronomia* of 1716 (and Euler in the 1740s). But the geometric mathematics used in the *Principia* offered no way of representing second derivatives.[6]

Thus in the *Principia* Newton did not write $F = ma = \frac{dp}{dt} = m\frac{d^2x}{d_2t^2}$. Thus Newton's second law means, $F \alpha (v - u)$ or $F = (v - u)$.

Also equations relating force with acceleration were given by Euler. The *Principia* involves geometry excessively. But the geometric mathematics used by Newton, and before him, did not at all represent acceleration and the second derivative. Thus Newton could have not written the equation $F =$

ma due to lack of mathematical basis. The dimensional analysis was started by Fourier in 1822[7,8] and dimensions of force are based upon $F = ma$ (given by Euler in 1775 not by Newton) and not on *Principia's* $F = (v - u)$. Thus the *Principia's* definition of the second law should have been discussed. Also the unit of force dyne[9] was initially defined in 1861, about 184 years after publication of the first edition of the *Principia*. But this definition was unacceptable to the Committee of the British Association for the Advancement of Science.[10] The 9th Conférence Générale des Poids et Mesures held in 1948 then adopted the name "newton" for unit of force in Resolution 7.[11] The concept of force was initially defined in 1687, so it took considerable time for development of laws and processes. Had the concepts of units, dimensions, acceleration, second derivative and related mathematical methods been available in Newton's time, then interpretation would have been different. It can be easily realized that the prevalent equation of force $F = ma$ was given by Euler as discussed below.

4. Euler Gave Four Equations of Force Related with Acceleration and Mass

Euler derived four equations of force—*i.e.* $F = ma/n$ in 1936,[12] $F = 2ma$ in 1750,[13,14] $F = ma/2g$ in 1765[15] and $F = ma$ in 1775.[16] Further if the vast literature of Euler (nearly 900 articles, scientific documents and books) is critically studied then more equations may be possible.[17] The equations were stated directly and independently at different times.

(ii) In 1716, Jacob Hermann published a Latin text called *Phoronomia*, meaning the science of motion.[18] He stated an equation $dc = pdt$, where c stands for "celeritas" meaning speed, and p stands for "potentia," meaning force or power, or:

$$p = \frac{dc}{dt} = \text{Force} \tag{3}$$

But in today's notation it is acceleration (rate of change of speed or celeritas), and is not force as described by the *Stanford Encyclopedia of Philosophy*.[6] However, Herman called it "potentia" or force or power at that time.

Further, momentum (P) was defined by *Jenning's Miscellanea*[19] in Latin in 1721 as

$$P = mv \tag{4}$$

In the next definition the velocity or speed is defined by dividing distance with time, *i.e.* S/t. These developments took place after the second edition (1713) and before the third edition (1726) of the *Principia*. And at this stage Newton could have conceptualized acceleration.[6] But Newton preferred not to mention it at all and kept original the second law of motion in tje third edition (1726), as in the first edition (1687) of the *Principia*.[6] Further some words and phrases in the *Principia's* second law were misinterpreted and the former was regarded as $F = ma$. The topic of the comparative study of developments of equations of motion of Newton and Euler has been recently studied in detail by various authors[20-24] including Raman.[5] But some issues further need to be discussed as Euler has written nearly 900 articles and books in the Latin and the French. All the works of Euler are not completely studied as of yet. Only 175 articles have been translated to English, thus analyzed by wider audiences. More useful results are expected if Euler's scientific literature is critically analyzed.[3]

Basically Euler has given four equations of force $F = ma/n$ (1736), $F = 2ma$ (1750), $F = ma/2g$ (1765) and $F = ma$ (1775) at different stages. Euler's various equations of force are completely independent of the *Principia's* second law of force, *i.e.* the alteration in motion is proportional to force *i.e.* $F = (v - u)$.

(a) In 1736, Euler wrote[12] equation of potentia (p) meaning force or power which has resemblance with Equation 4 *i.e.* $dc = pdt$,

$$dc = \frac{npdt}{m} \quad \text{or} \quad F = \frac{1}{n} ma \tag{5}$$

where m is mass, c is velocity, F is force, t is time and n is constant and depends upon unity of measure.[12,25] By unity of measure we mean unit of measurement. Euler[12] used two primary or fundamental units L (length) and F (force), thus coefficient/constant of proportionality is 2. Now co-efficients are determined experimentally. The systems of primary units L(length)-F(force)-T(Time) and L(length)-M(mass)-T(Time) were introduced in the following century. From Equation 6 Euler was able to derive all differential equations necessary to describe the motion of a point-mass.

(b) In *Mechanica*, however, Euler used an intrinsic coordinate system. He decomposed speeds and forces according to directions that depended upon the intrinsic nature of the problem. In these papers, Euler used an extrinsic reference frames (a system of three orthogonal Cartesian axes) and formulated the following equations of force[13-14]:

$$2Mddx = Pdt^2,\ 2Mddy = Qdt^2,\ 2Mddz = Rdt^2, \ \text{or}$$

$$P = 2M\frac{d^2x}{dt^2},\ Q = 2M\frac{d^2y}{dt^2},\ R = 2M\frac{d^2z}{dt^2} \tag{6}$$

where M is the mass and P, Q and R the components of the force on the axis (the coefficient 2 depended on the unity of measure, and Euler has chosen two primary units).

$$F = 2M\frac{d^2s}{dt^2} = 2M\frac{dv}{dt} = 2\frac{dp}{dt} = 2Ma \tag{7}$$

In view of dependence upon unity of measure coefficient n is 2 in Equation 6, *i.e.* $F = ma/2$. Raman[5] had only mentioned Equation 7 given by Euler whereas neglected other equations, *i.e.* Equations 5, 8 and 9 which are required to be mentioned for completeness and draw conclusions over a wide range. These equation are also given by Euler. Currently Equation 9, i.e. $F = ma$, is used as the equation of force. For completeness all equations have to be mentioned.

(c) Later, in 1765, Euler introduced the concept of moment of inertia of a rigid body and decomposed the motion into the rectilinear motion of the centre of mass and proposed equation[15]

$$F = \frac{Ma}{2g} \tag{8}$$

Further in 1775, Euler completed the construction of general equations of dynamics by formulating a system of six equations determining the motion of any body, which (except for an additional coefficient) he wrote in this way:[16]

$$P = \int dM \frac{d^2x}{dt^2},\ Q = \int dM \frac{d^2y}{dt^2},\ R = \int dM \frac{d^2z}{dt^2} \quad (9)$$

$$\int zdM \frac{d^2y}{dt^2} - \int ydM \frac{d^2z}{dt^2} = S,\ \int xdM \frac{d^2z}{dt^2} - \int zdM \frac{d^2y}{dt^2}$$

$$= T,\ \int ydM \frac{d^2x}{dt^2} - \int xdM \frac{d^2y}{dt^2} = U$$

Or in general, $F = \int dM \frac{d^2s}{dt^2}$

As $F = ma$ is the last or the simplest available equation given by Euler it is used in calculations. Further, $F = ma$ is used to derive rest mass energy equation ($E_0 = m_0c^2$), but if $F = 2ma$, $F = ma/2$ and $F = ma/2g$ are used then equations for rest mass energy become $E_0 = 2m_0c^2$, $E_0 = m_0c^2/2$ and $E_0 = m_0c^2/2g$. Similar is the situation for kinetic energy. The equation for *Principia's* Law II is $F = (v - u)$, which is not discussed at all. Thus $F = ma$ is an extensively used equation of force, which was given by Euler in 1775.

5. How Was Priority of *F = ma* Shifted to Newton from Euler?

Thus the last edition of the *Principia* in 1726 was not different from the first and second editions published in 1687 and 1713 as far as laws of motion are concerned. At this stage mathematical interpretation in terms of analytical and algebraic equations became a part of science. Thus scientists tried to express Newton's laws in mathematical equations. In 1775 Euler gave $F = ma$ 48 years after the death of Newton. This equation was extremely consistent with mathematical methods and easier to use in various phenomena. Thus scientists started using it extensively and it became indispensable (however, other equations of force were given by Euler). The text form of $F = ma$ is "the rate of change of momentum is proportional to impressed force."

This definition is based upon $F = ma$, and the *Principia's* second law of motion both involve force. This is perhaps the reason scientists simply replaced definition of Newton's second law of motion in the *Principia* with Euler's equation ($F = ma$). But it is completely inconsistent. The reason is that "motion" is replaced by "quantity (amount or quantum or magnitude) of motion." Also the word difference or change is regarded as equal to "rate of change of..." Also Euler's name is never associated with the law which was discovered by him.

About this aspect Truesdell remarked[5,26]: "Although these remarks were made over a decade ago, we still find textbooks in which $F = ma$ is called Newton's formula in which absolutely no mention of Euler in this context." It clearly implies that Euler's reference should be in textbooks, and logically all equations given by Euler be mentioned. At the same time it must be taken into account that Euler had given four equations of force and $F = ma$ has been chosen to explain the phenomena.

But the next question is when Euler's equation $F = ma$ was used as Newton's second law of motion. In view of it we considered two books published in 1871 and 1934; however it is added there may be many books. In a book,[27] *The First Three Sections of Newton's Principia*, it was noted that the Cambridge College and School textbook published in 1871 carries the second law of motion as stated in the *Principia*. Thus in a standard textbook a quote of the original law without distortion, even after 100 years of enunciation of $F = ma$. Thus misinterpretation of definition of the *Principia* took place after 1871. Whereas in another book[27] titled *Newton's Principia* published in 1934, the original form of the second law is quoted the same as in the *Principia*. But in an appendix an attempt was made to misinterpret the law taking motion and quantity of motion as the same also altering "rate of change of" as synonymous. It is assumed that $F = \Delta v = \frac{dp}{dt} = m \frac{d^2x}{d^2t^2}$, which is unscientific.

The text of a book by Cajori[28] gives the same reasons to change $F = (v - u)$ to $F = ma$ as given by Raman, which is already explained.

Historically the reason for this lapse may be that Euler's work was not well compiled as he worked in Switzerland, Germany and Russia, whereas Newton was based at the University of Cambridge and his work is well compiled in the *Principia*. Now Euler's work is well compiled by the Mathematical Association of America and available online.[17] Thus newer facts are coming in the picture. Then scientists found Euler's equation $F = ma$ exceptionally useful but may not be aware of actual originator, thus made arbitrary changes in the definition of the *Principia's* second law of motion so that it represents $F = ma$. Euler has independently given four equations of force.

Acknowledgements

The author is highly indebted to an anonymous reviewer for useful comments, and Prof. Robert Bradley and Prof. Erik Tou for critical discussions and encouragements at various stages of the work.

References

1. Hussey, E. 1983. *Aristotle Physics Book III and IV*, Oxford University Press.

2. Newton, I. 1727. *Mathematical Principles of Natural Philosophy*, pp. 19-20, translated by Andrew Motte; http://books.google.co.in/books?id=Tm0FAAAAQAAJ&pg=PA1&redir_esc=y#v=onepage&q&f=false

3. Sharma, A. 2014. Presented at The Euler Society Conference, July 21-23, St. Edward's University, Austin, TX.

4. Sharma, A. 2014. *Infinite Energy*, 21, 116, 38-40.

5. Raman, V.V. 1972. *The Physics Teacher*, 10, 136-137.

6. http://plato.stanford.edu/entries/newton-principia/#ThrEdiPri

7. Fourier, J. 1822. *The Analytical Theory of Heat*, Cambridge University Press.

8. Martins, R. 1981. "The Origin of Dimensional analysis," *J. Franklin Institute*, 331, 331.

9. Rossiter, W. 1879. *Dictionary of Scientific Terms*, William Collins, Sons, and Coy. p. 109.

10. Everett, ed. "First Report of the Committee for the Selection and Nomenclature of Dynamical and Electrical Units," 42nd Meeting of the British Association for the Advancement of Science, p. 223.

11. International Bureau of Weights and Measures. 1977. *The International System of Units* (330–331) (3rd ed.), U.S. Dept. of Commerce, National Bureau of Standards, p. 17.

12. Ferraro, G. 2010. "Euler's Analytical Program," *Quaderns d'història de l'enginyeria*, 11, 185-208.

13. Euler, L. 1750. "Découverte d'un nouveau principe de Mécanique," *Mém.*, 6, 185-217 or Opera, Ser. 2, Vol. 5, 81-108.

14. Euler, L. 1747. "Recherches sur le mouvement des corps célestes en général," *Mémoires de l'académie des sciences de*

Berlin, 3, 93-143 or Opera, Ser. 2, Vol. 25, 1-44.
15. Euler, L. 1765. *Theoria motus corporum solidorum seu rigidorum ex primis nostrae cognitionis principiis stabilita et ad omnes motus, qui in hujusmodi corpora cadere possunt, accommodata*, Rostochii et Gryphiswaldiae litteris et impensis A.F. Röse.
16. Euler, L. 1775. "Nova methodus motum corporum solidorum rigidorum determinandi," *Novi Commentarii academiae scientiarum Petropolitanae* (afterwards: Novi Comm.) 20, 208-238 or Opera, Ser. 2; Vol. 9, 99-125.
17. www.Eulersociety.org
18. Jacob, H. 1716. *Phoronomia, sive De viribus, motibus corporum solidorum et fluidorum libri duo*, R&G Wetstenios.
19. Jennings, J. 1721. *Miscellanea in Usum Juventutis Academicae*, Aikes & Dicey, p. 67.
20. Blay, M. 1992. *La naissance de la mtcanique analytique: La science movement au tourant des xviiie siecles*, PUF.
21. Blay, M. 1998. *Reasoning with the Infinite: From the Closed World to the Mathematical Universe*, University of Chicago Press.
22. Maronne, S. and Panza, M. 2010 "Newton and Euler," in *The Reception of Isaac Newton in the European Enlightenment,* Mandelbrot, S. and Pulte, H., eds., 2 vols., Continuum.
23. G Maltese, G. 2002. "On the changing Fortunes of the Newtonian Tradition in Mechanics" in *Two Cultural Essays in Honour of David Speiser*, Kim William, ed., Birkhauser, 97-113.
24. Sharma, A. 2014. *Physics Essays*, 27, 3, 503-509.
25. Guinness, G. 1994. *Companion Encyclopedia of History of Philosophy of the Mathematical Sciences*, Volume 2, The John Hopkins University Press, pp. 1011.
26. Truesdell, C. 1960. *Archive for History of Exact Sciences*, 1, 1, p. 23.
27. Evance, J.H. and Main, P.T. 1871. *The First Three Sections of the Principia*, Deighton, Bell and Co., pp. 215.
28. Cajori, F. 1934. *Newton's Principia*, University of California Press, p. 13.

About the Author

Ajay Sharma is Assistant Director in the Department of Education, Shimla (India). He began his career as a physics lecturer at DAV College Chandigarh (India). Sharma is the author of *Beyond Newton and Archimedes* and *Beyond Einstein and* $E = mc^2$. He has published over 50 research papers. His work is online at: http://www.AjayOnLine.us

*Fundamental Physics Society, Post Box 107, GPO Shimla 171001 HP, India
Email: ajoy.plus@gmail.com