Relativistic mass of secondary neutrons in nuclear fission

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Abstract: The relativistic variation of mass with speed is experimentally confirmed and widely used in physics. In fission reactions, neutrons have energy 0.025 eV or velocity 2185 m/s, whereas secondary neutrons have energy 2 MeV or 1.954×10^7 m/s: ∼7% speed of light. The mass of neutrons having classical velocity is 1.0086649 u and that of a neutron moving with relativistic velocity is 1.01080879 u (thus the increase in mass of a neutron is 0.212547% as the difference in mass is 0.00213 u). In calculations of the Q-value for fission reactions, the masses of relativistic or nonrelativistic neutrons are considered to be the same, i.e., 1.0086649 u, which contradicts the relativistic variation of mass. Consequently, the Q-value turns out to be 173.271 MeV. If a body moves with relativistic speed then its relativistic energy (hence relativistic mass) is taken into account. If the mass of primary neutrons is considered to be 1.0086649 u and that of secondary neutrons 1.01080879 u (actual values), then the Q-value turns out to be 167.29 MeV. So there is a variation of energy: 5.99 MeV or 3.45%. This implies that the efficiency of reaction is less in this case as Q-values decrease. Therefore, in calculation of energy in reaction, the relativistic mass of a neutron must be taken into account, whence the lower efficiency of reaction/reactor is explained. It is an established experimental observation reported in the literature over the past four decades that energy emitted in fission of U^{235} and Pu^{236} is 20–60 MeV less than the energy predicted by $\Delta E = \Delta mc^2$. This can be explained if speeds of other products are precisely measured and relativistic masses are calculated. These aspects of basic nuclear physics are not yet acknowledged, as the Q-value turns out to be lower. © *2015 Physics Essays Publication*.
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Résumé: La variation relativiste de la masse avec la vitesse est confirmée expérimentalement et largement utilisée en physique. Dans les réactions de fission, les neutrons ont une énergie de 0,025 eV ou une vitesse de 2,185 m/s, tandis que les neutrons secondaires ont une énergie de 2 MeV ou une vitesse de $1{,}954 \times 10^7$ m/s: ∼ 7% de la vitesse de la lumière. La masse de neutrons ayant une vitesse classique est de 1,0086649 u et celle d'un neutron en mouvement avec une vitesse relativiste est de 1,01080879 u (donc l'augmentation en masse d'un neutron est de 0,212547% car la différence en masse est de 0,00213 u). Dans les calculs de la valeur Q pour les réactions de fission, les masses de neutrons relativistes ou non relativistes sont considérées comme identiques, à savoir 1,0086649 u, ce qui est en contradiction avec la variation relativiste de la masse. Par conséquent, la valeur Q s'avère être de 173,271 MeV. Si un corps se déplace à une vitesse relativiste alors son énergie relativiste (donc masse relativiste) est prise en compte. Si la masse de neutrons primaires est considérée être de 1,0086649 u et que celle des neutrons secondaires est de 1,01080879 u (valeurs réelles), alors la valeur Q s'avère être de 167,29 MeV. Donc, il y a une variation de l'énergie: 5,99 MeV ou 3,45%. Ceci implique que le rendement de la réaction est inférieur dans ce cas vu que la valeur Q baisse. Par conséquent, dans le calcul de l'énergie en réaction, la masse relativiste d'un neutron doit être prise en compte, d'où la faible efficacité de réaction/réacteur s'explique. C'est une observation expérimentale établie rapportée dans la littérature au cours des quatre dernières décennies que l'énergie émise dans la fission de U^{235} et Pu^{236} est de 20-60 MeV et est inférieure à l'énergie prédite par $E = mc^2$. Cela peut s'expliquer si les vitesses des autres produits sont précisément mesurées et que les masses relativistes sont calculées. Ces aspects de la physique nucléaire fondamentale ne sont pas encore reconnues, vu que la valeur Q s'avère être plus faible.

Key words: Relativistic Mass; Neutron; Fission; Energy.

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I. FAST NEUTRONS ARE INITIALLY EMITTED IN NUCLEAR FISSION

The neutron was discovered by Chadwick[1] in 1932. Hahn and Strassmann reported nuclear fission in *Naturwissenschaften* and that a new element, barium, was obtained.[2] The discovery of fission and production of barium from uranium were also confirmed by Meitner and Frisch.[3,4] The mass of the neutron was measured by Chadwick at 1.0067 u, whereas the current mass of a neutron is 1.0086649156 u. In the fission of U^{235}, various fission fragments of different isotopes are possible. In the first ever controlled nuclear fission reaction, one of the fission fragments was barium; hence this reaction is considered in the following discussion.

In nuclear fission, the energy of a neutron is nearly 2 MeV (velocity 1.954×10^7 m/s); such neutrons cannot cause further fission. Thus, by means of a moderator, its speed is reduced to 0.025 eV (2185 m/s). The mass of a secondary neutron is then 1.01080879 u, not 1.0086649156 u (classical mass of the neutron). Thus the increase in mass of a secondary neutron is 0.00213 u or 0.215%, which cannot be neglected. The theme of this discussion is that the relativistic mass of a neutron has not been properly taken into account in the calculation of the Q-value. If it is taken into account then significantly new results are obtained; the Q-value turns out to be lower.

II. Q-VALUE AND RELATIVISTIC VARIATION OF MASS IN REACTION

The energy emitted in a nuclear reaction (or any other reaction) is given by

$$\begin{aligned} E &= \Delta m c^2 \\ &= [\text{mass of reactants} - \text{mass of products}]c^2 \end{aligned} \tag{1}$$

where Δm is the decrease in mass of reactants and c is the speed of light. Here, the mass of products means actual existing mass. If the product fragment has a relativistic speed then the mass must be taken as relativistic mass, not classical mass.

There are many reactants for which nuclear fission occurs and different fragments are obtained for each such reaction. For simplicity, we have the following typical nuclear fission reaction:

$$_{92}U^{235} + {_0}n^1 \rightarrow [_{92}U^{236}]^* \rightarrow {_{56}}Ba^{144} + {_{36}}Kr^{89} + 3{_0}n^1. \tag{2a}$$

The range of multiplicity of neutrons varies from zero to almost 10 (on average it is 2.5 for the fragment chosen). In this case, the mass number is conserved if the number of neutrons is taken as 3. The above reaction also proceeds in different way,[5]

$$\begin{aligned} _{92}U^{235} + {_0}n^1 \rightarrow [_{92}U^{236}]^* \rightarrow {_{54}}Xe^{140} + {_{38}}Sr^{94} \\ + {_0}n^1 + {_0}n^1. \end{aligned} \tag{2b}$$

The Q-value for the reaction is the amount of energy released in the reaction

$$\begin{aligned} \text{Q} - \text{Value} = [\text{mass of reactants}]c^2 \\ - [\text{mass of products}]c^2. \end{aligned} \tag{3}$$

A. Effect of relativistic variation of mass

The relativistic variation of mass with speed,[6–10] i.e.,

$$\text{M}_{\text{motion}} = \frac{M_{\text{rest}}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4}$$

would be irrelevant or vanish in the case of a particle moving with relativistic speed but with mass taken as classical. Equation (4) has been experimentally confirmed on numerous occasions but it is neglected in the cases described herein.

When the speed of the reactants or products is in the relativistic region then,

(i) mass decreases and hence mass is converted to energy;
(ii) mass increases due to a relativistic effect.

The effects are opposite in nature (increase or decrease in mass of fragments in the same reaction) and have to be taken into account. If either effect is neglected then conservation laws are not properly satisfied.

The Q-value depends upon the difference between the mass of reactants and products.

Further subcases are also considered depending upon speed of fission fragments.

1(a) If the speed of fission products is in the classical region, $v \ll c$, then,

$$\begin{aligned} \text{Q} - \text{Value} = [\text{mass of reactants}]c^2 \\ - [\text{mass of fission fragments when } v \ll c]c^2. \end{aligned} \tag{5}$$

1(b) If the speed of fission products is in the relativistic region, i.e., $v \sim c$ then,

$$\begin{aligned} \text{Q} - \text{Value} = [\text{mass of reactants}]c^2 \\ - [\text{mass of fission fragments when } v \sim c]c^2. \end{aligned} \tag{6}$$

When $v \sim c$, we cannot write

$$\begin{aligned} \text{Q} - \text{value} = [\text{mass of reactants}]c^2 \\ - [\text{mass of reactants when } v \ll c]c^2 \end{aligned}$$

When speed is in the relativistic region then mass increases. Thus, it can be safely concluded that the equation of relativistic variation of mass cannot be neglected. The various results are shown in Table I.

B. Magnitude of Q-value

1. Nonrelativistic neutrons

Thermal Neutrons (0.025 eV or 2185 m/s): If it is assumed that speed is in the classical region, i.e., $v \ll c$, then according to Eq. (3) the Q-value can be written as

TABLE I. The relativistic mass of a neutron for $v \sim c$.

S. No.	Type of neutron	Velocity	Mass
1	Primary	2185 m/s (0.025 eV)	1.0086649 u
2	Secondary	1.954×10^7 m/s $\sim 7\%$ c (2 MeV)	1.01080879 u

$$\text{Mass of reactants} = (235.0439299 + 1.0086649156)\text{u} = 236.0525948\,\text{u}. \tag{7}$$

The mass of a classical (primary) neutron is 1.0086649156 u. The mass of a relativistic (secondary) neutron is regarded as 1.0086649156 u

$$\text{Mass of products} = (143.922953 + 88.917630 + 3.02599473)\text{u} = 235.8665777\,\text{u}; \tag{8}$$

$$\text{Mass annihilated} = \Delta m = 236.0525948\,\text{u} - 235.8665777\,\text{u} = 0.18601714\,\text{u}. \tag{9}$$

$$\text{Energy released (Q-Value)} = 173.271\ \text{MeV}. \tag{10}$$

2. Relativistic neutrons

Fast Neutrons (2 MeV or 1.954×10^7 m/s): The neutrons which are emitted in the fission are fast (secondary) neutrons, having energy nearly equal to 2 MeV (3.2×10^{-13} J). At this energy, a neutron moves with relativistic speed, i.e., 1.954×10^7 m/s ($\sim$7% that of light). The mass of a neutron moving with relativistic speed is 1.010808793 u (whereas its rest mass is 1.0086649156 u). If the speed is within relativistic limits then the particle mass increases,[6–10] according to Eq. (4). The relativistic effects cannot be neglected for otherwise it would mean that

$$\begin{aligned} &\text{M}_{\text{motion}}\left(\text{at } 1.954 \times 10^7\ \text{m/s}\right) = \text{M}_{\text{rest}} \\ &\text{or } 1.010808793\,\text{u} = 1.0086649156\,\text{u} \end{aligned} \tag{11}$$

which is not justified. Thus when neutrons move with relativistic speed, relativistic mass has to be taken into account.

The relativistic mass of a neutron can be calculated from the equation for relativistic kinetic energy

$$\begin{aligned} &\text{K} = [\text{M}_\text{m} - \text{M}_\text{r}]c^2, \\ &\text{M}_\text{m} = \text{K}/c^2 + \text{M}_\text{r} = 3.204 \times 10^{-13}/9 \times 10^{16} + \text{M}_\text{r}, \\ &\text{M}_\text{m} = 0.002143883\,\text{u} + 1.0086649156\,\text{u} = 1.010808793\,\text{u}. \end{aligned} \tag{12}$$

Hence, the mass of fast neutrons must be 1.01080879 u (an increase of 0.2125%), i.e., more than the rest mass 1.0086649156 u. This cannot be neglected. If a body moves with relativistic speed then its relativistic energy (hence relativistic mass) must be taken into account. The mass of the product neutron (2 MeV, 1.954×10^7 m/s, $\sim$7% that of light) must be different from the reactant neutron (0.025 eV, 2185 m/s). Now substituting these values into Eq. (3), in this case the Q-value further decreases as the mass of products is higher

$$\text{Mass of reactants} = 236.0525948\,\text{u}. \tag{13}$$

The exact relativistic mass of secondary neutrons is 1.010808793 u

$$\text{Mass of products} = (143.922953 + 88.917630 + 3.032426394)\,\text{u}, = 235.8730094\,\text{u}. \tag{14}$$

$$\begin{aligned} \Delta m &= 0.1795858\,\text{u}, \\ \text{Q} &= 167.28\,\text{MeV}. \end{aligned} \tag{15}$$

Hence, when relativistic mass of neutrons is considered the energy predicted is 5.99 MeV (2.29×10^{-13} J) less. Now the difference in mass in both cases (when relativistic and classical masses of neutrons are considered) is 0.0064317 u. So the energy predicted is overvalued by 3.45% when the classical mass of the neutron is used, even when it moves with speed comparable with c. Such relativistic effects need to be included for the other fission fragments.

When the relativistic speed of a neutron is measured, its mass increases, hence the Q-value decreases by 6 MeV. Now if masses of other fission products, i.e., Ba^{144}, Kr^{89}, $_{54}\text{Xe}^{140}$, $_{38}\text{Sr}^{94}$, etc., are measured and found in the relativistic region, then mass will increase, thus energy would be even less.

In this regard, experiments involving Relativistic Radioactive Beams utilizing secondary beam facilities as available at GSI are useful.[11] In these experiments, the velocity and total kinetic energy of fission fragments are measured along with other parameters. With such experiments or specifically improved experiments, the speed, hence the relativistic mass, of the fission fragments, can be calculated. In the present consideration, the product fragments are Ba^{144}, Kr^{89}, $_{54}\text{Xe}^{140}$, $_{38}\text{Sr}^{94}$, etc. If the speed is found in the relativistic region, then mass would be greater, hence energy emitted will be less. In fission of U^{235} and Pu^{239} energy is found experimentally to be 20-60 MeV less.[12–14] If the products have speeds comparable with c, then mass would be greater so that mass defect and energy would be smaller. The lower energy observed can therefore be explained. Some other methods can also be employed to explain it.

The exact measurements of relativistic masses and energies are required in calculations as in Eq. (12). The precise, specific, and independent experimental measurements of energy emitted in a single fission event will be helpful in this regard. It would be better if this value is used as the standard for understanding such reactions. At large Hadron collider, the experiments were conducted at energy level of 7 TeV, then proton attained speed equal to 0.99999997 time that speed that of light and temperature was 10^{16}°C (temperature more than 1×10^9 times greater than prevailing at the center of the Sun). Thus mass of proton would increase considerably at this stage and need to be properly assessed in view of Eq. (4) before drawing conclusions. Here, practically the speed of proton and its temperature is exceptionally high, as mentioned above, so exact relativistic mass of proton must be taken in account, not the rest mass in further interpretation. If it is not so, then it would mean contradiction of relativistic variation of mass, i.e., Eq., (4), which is widely confirmed experimentally. In new experiments, the energies are increased to 13–14 T eV, thus both speed and mass of protons (particles) would be further higher.

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[1]J. Chadwick, Nature **129**, 312 (1932).
[2]O. Hahn and F. Strassmann, Naturwissenschaften **27**, 11 (1939).
[3]L. Meitner and O. R. Frisch, Nature **143**, 239 (1939).
[4]O. R. Frisch, Nature **143**, 276 (1939).
[5]A. Beiser, *Concepts of Modern Physics*, 5th ed. (Tata McGraw-Hill, New Delhi, 1995), pp. 451–454.
[6]E. Segre, *Nuclei and Particles*, 2nd ed. (W.A. Benjamin, Reading MA, 1977), p. 591.
[7]H. A. Lorentz, Proc. R. Soc. Amsterdam **1**, 427 (1899).
[8]H. A. Lorentz, Proc. R. Soc. Amsterdam **6**, 809 (1904).
[9]W. Kaufmann, Phys. Z. **4**, 54 (1902).
[10]A. H. Bucherer, Ann. Phys. **28**, 513 (1909).
[11]K. H. Schmidt, S. Steinhäuser, C. Böckstiegel, A. Grewe, A. Heinz, A. R. Junghans, J. Benlliure, H. G. Clerc, M. de Jong, J. Müller, M. Pfützner, and B. Voss, Nucl. Phys. A **665**, 221 (2000).
[12]E. G. Bakhoum, Phys. Essays **15**, 87 (2002)
[13]F. J. Hambsch, H. H. Knitter, C. Budtz-Jørgensen, and J. P. Theobald, Nucl. Phys. A **491**, 56 (1989).
[14]H. Thierens, A. De Clercq, E. Jacobs, D. De Frenne, P. D'hondt, P. De Gelder, and A. J. Deruytter, Phys. Rev. C **23**, 2104 (1981).