Akhlesh Lakhtakia
Professor
Engineering Science and Mechanics
The Pennsylvania State University
227 Hammond Building
University Park, PA 16802-1401
Tel: (814) 863-4319 Fax: (814) 863-7967
E-mail: AXL4@PSU.EDU
PENNSTATE

Best wishes

Speculations in Science and Technology **20**, 297–300 (1997)

A generalization based on quantitative inadequacy of Archimedes principle in balloon experiments

AJAY SHARMA

Community Science Centre, Post Box 107, Directorate of Education, Shimla 171001 (HP), India

One consequence of Archimedes' principle is that the mass which a balloon supports in a fluid is independent of the shape of the balloon and depends only upon its volume. For air-filled floating balloons in water some deviations from this result have been observed in first-stage experiments. The dependence of mass on the shape of the balloon which is supported in water has been clearly observed in various observations. It is evident from the first-stage experiments that for floating balloons the principle is only true for particular shapes. The deviations from the principle and contradictions can be explained if the definition of the principle is empirically modified i.e. it is assumed that the upthrust experienced by body is proportional to the weight of the fluid displaced. The constant of proportionality also accounts for shape of body and other relevant factors that were not accounted for by the principle.

Keywords: Archimedes' principle; balloons.

Introduction

Archimedes' principle states that when a body is immersed in a fluid it experiences an upthrust (or buoyant force) and its weight decrease is equal to the weight of fluid displaced [1]. Archimedes (287–212 BC) stated this principle about 2200 years ago (ca. 250 BC). The mathematical equations based upon the principle became derivable about 1935 years after the enunciation of the principle when Newton published the law of gravitation in the *Principia*. It is very difficult to understand how scientists took the principle for granted for about 1935 years without mathematical backing and treated it as an established law. It is now used as one of the methods to determine the densities of bodies [2]. However, the densities can be measured most accurately by direct methods [1].

Floating balloons

Some experiments on floating balloons have not been conducted. One of the reasons may be that relevant mathematical equations became derivable only about 1935 years after enunciation of Archimedes' principle. Consider a balloon in which volume V is filled with air of density D_a and is floating in water of density D_w supporting a mass m_f. Then the total weight is

$$W = (VD_a + m_f)\, g \tag{1}$$

where VD_a is the mass of air filled in the balloon; m_f is sum of masses of sheath of balloon and the additional mass attached for balancing the balloon. According to Archimedes' principle an upthrust experienced by the balloon in water is

Now Springer

$$U = (V + v_f) D_w g \tag{2}$$

where v_f is sum of volumes of sheath and that of the additional mass, if any, attached to balloon i.e. the volume corresponding to m_f. When the balloon floats, one has $W = U$ or equivalently,

$$m_f = (V + v_f) \; D_w - VD_a \tag{3}$$

A similar mathematical treatment can be found in Borowitz and Beiser [3], but they have neglected v_f which is very significant.

To conduct the experiments properly the value of v_f can be kept constant and determined in the following way. For this, a small pot of non-hygroscopic material fitted with a hook and lid should be fabricated. With the help of the hook the small pot can be suspended from a non-hygroscopic balloon or vessel (which also has a hook). To balance the balloon or vessel, the masses can be placed inside the small pot opening the lid carefully outside the water. The masses should be in the form of fine granules or powder because in order to balance the balloon properly, masses of the order of a fraction of milligram may be required. The mass which the balloon supports should be the mass of the pot and the mass of the vessel sheath. The rest of the terms can be measured easily.

Experimental verification of Equation 3

Some experiments were conducted for about 6 months in 1991 on air filled balloons of different shapes. According to the principle m_f depends only upon the volume of the balloon, not on its shape. From these first-stage experiments, the dependence of m_f on the shape of balloon was clearly observed. The volume of balloons varied from the nearly 175 cm^3 to 1000 cm^3. For the balloons of volume 175 cm^3 to about 550 cm^3, the shapes varied from spherical to cylindrical; and the deviations ranged from 3 to 1.5%. Next when the volume was increased from 550 cm^3 to about 1000 cm^3; the shape of the balloon varied from cylindrical to nearly spherical and the percentage deviations increased from 1.5 to 3.

In the first-stage experiments, the general trend of deviation i.e dependence of m_f on the shape of balloon remained the same. The magnitude of deviations was considerably and reasonably higher for balloons of spherical shape and lower for those with elongated shapes. This magnitude (of deviations) is expected to be higher for umbrella-shaped balloons and lower for long pipe shaped. Apparently, from first-stage experiments it is very clear that Archimedes' principle applied to floating balloons immersed in water under near equilibrium conditions will be true only for a particular shape. However, to confirm the magnitude of the deviations precise tests with the most sensitive equipments are required.

Deviations and speculative generalisation of Archimedes' principle

The deviations from the principle are expected in precise measurements or observed in first-stage experiments mainly on the basis of two reasons i.e. the following are the factors which can influence the results but were not taken in account in deriving the principle.

(1) Shape of balloon: according to Equation 3, m_f depends only upon volume of balloon and not on its shape. If the internal volume of balloon or vessel is 1000 cm^3 and volume of sheath is 100 cm^3. Now it can have any shape e.g. spherical, umbrella shaped, long pipe shaped or

distorted shaped. In all cases, the volume of balloon or vessel remains the same, hence m_f should be the same. But it is evident from the first-stage experiments that m_f also depends upon the shape. So some sensitive experiments are required for confirmation. For balloons or vessels of different shapes, the thrusts on the bases (pressure × area of cross-section of base) will be different.

(2) Depth and magnitude: according to the principle, m_f is independent of the depth at which the balloon floats in the tank of water. It means that the balloon may float at upper surface in the middle of tank or at the bottom; and the m_f should remain the same. But it may not be true on the basis of the following established fact. If a submarine is to be lowered to an increasing depth, then more water (equivalent to an additional mass in the case of the balloon) has to be put into the tanks. So it can be concluded that balloon may support slightly more mass at the bottom than at the upper surface of the water. It is observed in the first-stage experiments that the effect of depth is not as strong as that of shape in such experiments. Also according to Equation 3, m_f remains the same irrespective of the fact that the balloon floats in a bucket, drum or tank of water at any position if the density of water remains the same. So some very sensitive experiments are required, as it is observed in first-stage experiments that this effect is as weak as that of the depth.

Effect of density of additional mass attached

According to Equation 3, if $1000\,cm^3$ of air of density $0.001293\,g\,cm^{-3}$ fills a balloon then it will float in water of density $1\,g\,cm^{-3}$ supporting mass equal to 1,098.707 g if v_f is $100\,cm^3$. In the case that the mass which the balloon of any arbitrary shape supports (m_f) is found to equal 1099.8057 g or 1097.6083 g, then the percentage deviation from the principle will be 0.1. In the case that the vessel is completely evacuated then from Equation 3 the value of m_f turns out to be 1100 g.

Furthermore, it is also evident that v_f (the volume of the sheath and masses) is quite significant in the calculation of m_f. The magnitude of v_f varies with the density of the masses used. For example, 1 kg each of aluminium ($2.7\,g\,cm^{-3}$), silver ($10.5\,g\,cm^{-3}$) and platinium ($21.5\,g\,cm^{-3}$) have volumes 370.3703, 95.238 and $46.5462\,cm^3$ respectively. So if additional masses of aluminium, silver, and platinium are used for balancing the same balloon then m_f will be different. Even glycerine which has a density of $1.26\,g\,cm^{-3}$ can be used as the mass but for this, the pot to be fabricated must be quite large. So it is concluded that even if the principle holds good if the additional masses used are of aluminium then it may not hold if the additional masses used are of silver or platinium. So, in all cases, experiments have to be conducted separately for balloons of different shapes using different masses; in various fluids of high or low density and viscosity.

For appreciable deviation from the principle the lighter or less dense masses (higher v_f, thus higher m_f) should be used if balloon or vessel is pipe shaped. Further, for spherical or umbrella shaped balloons the heavier or dense masses (less v_f) should be used, in this case m_f will be less according to Equation 3. If the balloon has a spherical shape, then in first-stage experiments the mass which balloon actually supports is more often found. Furthermore, to understand the effect of v_f, the dense masses can be added to the inside of the balloon or the vessel, instead of adding them to the pot. In this case, the balloon or vessel can be judiciously fabricated and should be fitted with a lid. Thus, the total volume of air inside the vessel will be the internal volume of the vessel less the volume of the mass added. If in the conducting experiments, the balloons or vessels are umbrella shaped, then reasonably high deviations would be expected.

The generalised form of the principle

In order to understand and summarise the experimental deviations from the expected consequences of the Archimedes' principle one may make a generalisation of the principle which will be valid for balloon experiments in near equilibrium conditions. It must be understood the conditions are far from ideal and not strictly hydrostatic. Small scale motion of the fluid and deformation of the balloon must be taken into consideration. Thermodynamical considerations and the possible presence of convection cannot be ignored.

One may conclude that such deviations can only be explained if the enunciation of the principle is modified i.e. if one assumes that upthrust (or buoyant force) experienced by balloon is proportional to the weight of fluid displaced by the body. Hence. Equation 2 becomes.

$$U = f(V + v_{\mathrm{f}})D_{\mathrm{w}}\, g \tag{4}$$

where f is a dimensionless constant of proportionality which is not equal to unity in all cases, and its value can be determined experimentally. The modified form of the principle as stated in Equation 4 is thus capable of explaining the deviations in the first-stage floating balloon experiments. The value of f is such that the mass which the principle theoretically predicts is equal to the mass that the balloon experimentally supports. Further, as f is dimensionless it is expected to depend upon the shape of balloon and also on the dimensionless ratios v_{f} / V and $D_{\mathrm{a}} / D_{\mathrm{w}}$ and other relevant factors. Finally, we must re-emphasise that some sensitive experiments are required to confirm such a dependence.

Acknowledgements

The author is highly indebted to Professor S.P. Puri. Dr D.L. Sharma and many others for constant encouragement. The constructive suggestions from Professor P. Khastgir and financial assistance from Community Science Centre, Shimla is also gratefully acknowledged.

References

1 Abbott, A.F. (1970) *Ordinary Level Physics*, pp. 112–7 and 138–47. New Delhi: Heinemann Educational Books.

2 Green, C.R. (1984) *Technical Physics*, pp. 217–31. Englewood Cliffs: Prentice Hall.

3 Borowitz, S. and Beiser, A. (1967) *Essentials of Physics*, pp. 203–10. California: Addison Wesley Publishing Company.

Paper received 25 November 1994; accepted 12 February 1996 (Communicated by P. Khastgir)