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THEORETICAL DEDUCTIONS FAVOUR THE GENERALISED FORM OF ARCHIMEDES' PRINCIPLE IN CASE OF COMPLETELY SUBMERGED FLOATING BALLOONS

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Earlier in case of completely submerged floating balloons the original form of Archimedes' principle has been generalised mainly due to reason that it does not account for ***shape of floating body*** *and* ***coefficient of viscosity of medium*** *in which body floats. In the generalisd form of Archimedes' principle upthrust must be regarded as proportional to (instead equal to) the weight of fluid displaced. There are concrete theoretical evidences that in case of completely submerged floating balloons under some particular cases the original form of Archimedes' principle is not mathematically applicable. It means that the volume V of medium filled in balloon can not be calculated as we get indeterminate form. However in this case the generalised form of Archimedes' principle predicts exact volume V. Thus the generalised form is applicable even in those cases when original form of the principle is not applicable, thus generalisation of the principle enhances its region of applicability.*

INTRODUCTION : In critical analysis of completely submerged floating balloons Archimedes' principle has been generalised *i.e.* upthrust is regarded as proportional to the weight of fluid displaced [1]. Thus for completely submerged floating balloon the generalised upthrust is given by

$$U = C\,(V + v)\,D_m\,g \qquad \ldots(1)$$

where V is volume of fluid filled in balloon, v is volume of sheath of balloon including if any external mass is attached to it, D_m is density of medium in which balloon floats. Here C is constant of proportionality (like Hubble's constant or coefficient of thermal conductivity) which accounts for the shape of body, viscosity, magnitude, convectional currents and surface tension of medium etc.

Archimedes stated the principle in 250 BC and mathematical equations became feasible in 1685 (when Newton published the **Principia** and defined acceleration due to gravity, g) *i.e.* after 1935 years. Archimedes' principle is used for determination of densities of bodies of irregular shapes. The volume of body is given by (2)

$$V = (W - w)/D_m\,g \qquad \ldots(2)$$

where W is weight of body of volume V in vacuum, w is weight of body in fluid of density D_m may be of high co-efficient of viscosity (co-efficient of viscosity of glycerine is 1058 times more than that of water at 20^{o}C and density).

But it is interesting to note that magnitude of 1 cc (unit of volume in CGS system) has undergone different phases (3). In 1889,

$$1 \text{ cc} = 1 \text{ ml} \qquad \ldots(3)$$

In 1907, 1 cc was regarded as

$$1 \text{ cc} = 0.999972 \text{ ml} \qquad \ldots(4)$$

In 1964, 1 cc was again redefined equal to 1 ml *i.e.*

$$1 \text{ cc} = 1 \text{ ml} \qquad \ldots(3)$$

Now it is difficult to understand that how Archimedes' principle was verified between 250 BC–1685 i.e. for 1935 years without mathematical equations; then for 204 years i.e. between 1685–1889, when there was no definite unit of volume. Further %age difference between Eq. (3) and Eq. (4) is 2.28×10^{-3}. Thus there has been error 2.28×10^{-3}% in measurement of volume by Eq. (2) either between 1907–1964 or after 1964.

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Anyhow with this developmental background we can critically and mathematically analyse Archimedes' principle in other aspects and further justify the generalised form of Archimedes' principle.

THEORETICAL JUSTIFICATION OF EQ. (1) IN SOME PARTICULAR CASES :
According to original form of Archimedes' principle, the mass which a balloon filled with medium of density d floats in water of density D is given by (1)

$$m = (V + v) D - Vd \quad \ldots(5)$$

From Eq. (5) different values of V, D, d and v can be written as

$$V = (m - vD)/(D - d) \quad \ldots(6)$$

$$D = (m + Vd)/(V + v) \quad \ldots(7)$$

and

$$d = \{D (V + v) - m\}/V \quad \ldots(8)$$

$$v = \{m - V (D - d)\}/D \quad \ldots(9)$$

Let internal volume of balloon or vessel is 200 cc filled with platinum (21.5 gm per cc) of volume 5.5 cc, silver (10.5 gm per cc) of volume 8.8 cc and remaining 85.7 cc is filled with aluminium 2.7 gm per cc, CO_2 and air such that its combined density is equal to 1 cc. Let volume of sheath is 20 cc and made of non-hygroscopic material and floats in water. The sheath may be non-hygroscopic if its composition is 93.8% in weight Nylon 12 and 6.2% Santicizer [4]. Now in this case $V = 200$ cc, $v = 20$ cc, $d = D = 1$ gm per cc; then mass of sheath can be calculated from Eq. (5)

$$m = (V + v) D - VD = (200 + 20) - 200 = vD = 20 \text{ gm} \quad \ldots(10)$$

It is evident that when $D = d$, then mass of sheath of volume 20 cc must be 20 gm. Thus in this case mass which balloon supports is independent of volume V *i.e.* volume of medium (Pt, Ag, Al, CO_2 and air). But it can be justified with help of logical deductions that this interpretation is not correct or logical.

(i) Firstly independence of mass on volume V as in Eq. (10), may be regarded as that it is true for every volume which may be 0.2 cc or less and 2×10^6 cc or more.

If volume V of medium (Pt, Ag, Al, CO_2 and air) filled inside is 2×10^6 cc. Then this magnitude of medium can never be contained in sheath of volume 20 cc and mass 20 gm. Obviously the sheath can never be able to support volume 2×10^6 cc and hence will break down. The constituents Pt, Ag and Al fall down; CO_2 and air will rise upward. Thus it does not represent actual experimental situation that balloon floats completely submerged in fluid.

Now if volume V may be regarded as true for every volume then it may be 2.1×10^8 cc, 2×10^{10} cc or 0.2 cc. Thus for this particular set up ($D = d = 1$ gm per cc, $V = 200$ cc, $v = 20$ gm) Eq. (10) based upon original form of Archimedes' principle predicts numerous values of V which is not correct. Also in numerous predicted cases the identity of balloon may vanish. Thus the numerous values can not be assigned to V which is 200 cc. Thus original form of Archimedes' principle is not applicable in this particular case.

However, it is not so in case the generalised form of Archimedes' principle is used, even under this particular case (when original form is not applicable). In view of the generalised form *i.e.* Eq. (1), the Eq. (5) can be written as

$$M = C (V + v) D - Vd \quad \ldots(5a)$$

If, $D = d = 1$ gm per cc then

$$M = C (V + v) - V = (C - 1) V + vC = (C - 1) 200 + 20C \quad \ldots(10a)$$

Thus now the mass which balloon supports depends upon the volume V and value of C has to be determined experimentally.

If we try to find out volume V from original form of Archimedes' principle under the condition *i.e.* $D = d = 1$ gm per cc then Eq. (6) with help of Eq. (10) becomes

$$V = (vD - vD)/(D - D) = 0/0 \quad \ldots(11)$$

which is indeterminate form *i.e.* volume becomes undefined but in actual experimental set up volume is V (*i.e.* 200 cc). Thus RHS of Eq. (11) becomes devoid of units and dimensions which is meaningless. Although division by zero is not permitted, yet is smoothly follows from experimental requirements when Archimedes' principle is applicable as per its definition. Also in this case numerator of the equation also becomes zero. It implies that in this case principle is not applicable. In determination of D, d and v in all terms V is required. As V in Eq. (11) is indeterminate form so these terms can not be calculated from original form of the principle. Thus Archimedes' principle in original form; in this particular case is not applicable. The similar results can also be achieved in case of other fluids like glycerine and mercury.

For better conceptual understanding a logical example can be quoted; the law of gravitation involves five terms as in case of Eq. (5). If four terms are given and law of gravitation is applicable then unknown term can be calculated so it is regarded as complete law. But it is not so in case of Archimedes' principle in this particular case. However under these conditions the generalised form of Archimedes' principle gives exact results *i.e.* volume V. Thus it is obvious that under those conditions when the original form of Archimedes' principle is not applicable, under those conditions the generalised form of Archimedes' principle is applicable. Also unlike Eq. (10), in Eq. (10a) the mass depends upon the volume V.

Now in view of the generalised form of Archimedes' principle Eq. (6–9) can be written as

$$V = (M - CvD)/(CD - d) \quad \ldots(6a)$$

$$D = (M + Vd)/C\,(V + v) \quad \ldots(7a)$$

$$d = \{CD\,(V + v) - M\}/V \quad \ldots(8a)$$

$$v = \{m - V\,(CD - d)\}/CD \quad \ldots(9a)$$

When $D = d$, then mass M is given by Eq. (5a), then volume can be calculated from Eq. (6a),

$$V = (C - 1)\,V/(C - 1) = V \quad \ldots(11a)$$

Thus in case of generalised form of Archimedes' principle neither numerator nor denominator become zero exactly under those conditions when original form of Archimedes' principle gives indeterminate form for volume V (Pt, Ag, Al, Co and air). This is an advantage of the generalised form of the principle over its original form. Hence in similar way we also get,

$$D = d \quad \ldots(12)$$

$$d = D \quad \ldots(13)$$

$$v = v \quad \ldots(14)$$

which is true. Hence it is theoretically confirmed that the generalised form of Archimedes' principle enhances the utility of Archimedes' principle in case of completely submerged floating balloons under this particular situation also.

EXPERIMENTS TO CONFIRM THE GENERALISED FORM OF ARCHIMEDES' PRINCIPLE : It is evident that the generalised form of Archimedes' principle has been theoretically justified as in Eqs. (11a, 12, 13 and 14) when exact and logical results are obtained. The same may also be experimentally confirmed if experiments with umbrella shaped bodies are specifically and quantitatively conducted in fluids of high viscosity and density. It is obvious that the coefficient of viscosity for glycerine at 20°C is about 1058 times more than that of water whereas its density is only 1.26 times than that of water. But upthrust is independent of coefficient of viscosity. It is stressed that specific and sensitive experiments may be conducted to check effects of **shape of bodies and coefficients of viscosity of media**. Both these factors have been observed to play significant roles in case of falling and rising bodies; and are taken in account in with help of Stokes law. So far these two factors have not been quantitatively studied specifically in case of completely submerged floating balloons or in similar other cases.

(i) Experiments in glycerine : Let us fabricate a vessel (balloon) which has internal volume 200 cc and volume of sheath is equal to 20 cc. Let internally vessel is filled with suitable amounts of platinum, silver, aluminium, Co and air such that its overall density of inside medium (*i.e.* d) is equal to 1.26 gm per cc. Now according to original form of Archimedes' principle *i.e.* from Eq. (10) (equivalently written for this situation) mass of the sheath should be 25.2 gm (thus density of sheath is also equal to 1.26 gm per cc).

Now, let a vessel ($V = 2{,}00$ cc; $v = 20$ cc, $m = 25.21$ gm) of arbitrary shape (typically flat or umbrella shaped) floats in glycerine (may be cup, jug or tank such that its density is 1.26 gm per cc) at any depth. This will be deviation from original form of Archimedes' principle. The value of C will be 1.000036075 from Eq. (5a). Thus %age deviation from Archimedes' principle will be 3.6×10^{-3}.

(ii) Experiments in mercury : Let us fabricate vessel which has internal volume 200 cc and volume of sheath is equal to 20 cc. Let vessel is filled with suitable amount of medium as in previous case such that its overall density of the medium inside is equal to 13.6 gm per cc. Now according to original form of Archimedes' principle *i.e.* from Eq. (10) (equivalently written for this situation) the mass of sheath should be 272 gm per cc (such that density of sheath should be 13.6 gm per cc.)

Now let a vessel ($V = 200$ cc, $v = 20$ cc and $m = 272.5$ gm) of any arbitrary shape floats in mercury (may be in cup, jug or tank) of density 13.6 gm per cc experimentally. Then value of C from Eq. (5a) will be 1.000167. Thus %age deviation from original form of Archimedes' principle is 1.67×10^{-2}. However all observations are consistent with the generalised form of Archimedes' principle. Similarly many more such tests are possible which may support the generalised form of Archimedes' principle. Thus some specific experiments are required for experimental confirmation of the generalised form of Archimedes' principle. Theoretically the comparison between original and generalised forms of Archimedes' principle is shown on the Table 1.

Table 1 : The comparison of original and the generalised forms of Archimedes' principle.

Original form	The generalised form
(i) Theme of definition : Upthrust is **equal** to the weight of fluid displaced.	(i) Theme of definition : Upthrust is **proportional** to the weight of fluid displaced.
(ii) Mathematical equation, $u = VD_m g$ where V is volume of body, and D_m is density of fluid.	(ii) Mathematical equation, $U = CVD_m g$ where C is constant of proportionality its nature is like Hubble's constant or coefficient of viscosity
(iii) It is valid if fluid and body are precisely at rest.	(iii) It is valid even if there is motion of body or fluid.
(iv) Condition of floatation for a completely submerged body (precisely at rest) $D_b = D_m$ where D_b is density of body.	(iv) Condition of floatation for a completely submerged body (not necessarily at rest) $D_b = CD_m$ Thus density body may or may not equal to that of medium.
(v) In floating bodies only D_m and D_b are relevent :	(v) Besides densities, shape of body viscosity, surface tension, motion and magnitude of medium may be equally significant.
(vi) In some cases the volume of medium filled in balloon can not be calculated.	(vi) In all cases the volume of medium filled in balloon hence other terms can be precisely determined.
(vii) Mathematically it is special case of generalised form if value of C is unity.	(vii) It is general formulation; some experiments have been suggested to confirm it.

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