Is $E = \Delta mc^2$ mathematically derived or speculated in Einstein's September 1905 paper?

Ajay Sharma[a)]
Fundamental Physics Society, His Mercy Enclave, Post Box 107 GPO, Shimla 171001, HP, India

(Received 21 May 2012; accepted 3 September 2013; published online 30 December 2013)

Abstract: In his paper, Einstein derived $\Delta L = \Delta mc^2$ (light energy–mass equation). It has not been completely studied; it is only valid under special conditions of the parameters involved, e.g., number of light waves, magnitude of light energy, angles at which waves are emitted and relative velocity v. Einstein considered just two light waves of equal energy, emitted in opposite directions and the relative velocity v uniform. There are numerous possibilities for the parameters which were not considered in Einstein's derivation. $\Delta E = \Delta mc^2$ is obtained from $\Delta L = \Delta mc^2$ by simply replacing L by E (all energy) without derivation. Fadner [Am. J. Phys. **56**, 144 (1988)] correctly pointed out that Einstein neither mentioned E or $\Delta E = \Delta mc^2$ in the derivation. Herein additional results are critically analyzed, taking all possible variables into account. Under some valid conditions of parameters, $\Delta L = \Delta mc^2$ is not obtained, e.g., sometimes the result is $M_a = M_b$ or no equation is derivable. If all values of valid parameters are taken into account, then the same derivation also gives $L \propto \Delta mc^2$ or $L = A\ \Delta mc^2$, where A is a coefficient of proportionality. Thus, Einstein's derivation under valid parameters also predicts that energy emitted may be less than or more than $\Delta L = \Delta mc^2$. © *2013 Physics Essays Publication.*
[http://dx.doi.org/10.4006/0836-1398-26.4.509]

Résumé: Dans son article Einstein a dérivé $\Delta L = \Delta mc^2$ (équation énergie de la lumière-masse). Il n'a pas été complètement étudié, il n'est valable que dans des conditions particulières des paramètres intervenants, par exemple nombre d'ondes de la lumière, grandeur de l'énergie lumineuse, angles d'émission des ondes, et vitesse relative v. Einstein a considéré seulement deux ondes lumineuses d'égale énergie, émises dans des directions opposées et la vitesse relative v uniforme. Il existe de nombreuses possibilités pour les paramètres qui n'ont pas été pris en compte dans la dérivation d'Einstein. $\Delta E = \Delta mc^2$ est obtenu à partir de $\Delta L = \Delta mc^2$ simplement en remplaçant L par E (toute l'énergie) sans dérivation. Fadner [Am. J. Phys. **56**, 144 (1988)] a bien souligné que Einstein ni mentionne E ou $\Delta E = \Delta mc^2$ dans la dérivation. Ici des résultats additionnels sont analysés en critique, en prenant en considération toutes les variables possibles. Sous certaines conditions valides des paramètres on n'obtient pas $\Delta L = \Delta mc^2$, par exemple parfois le résultat est $M_a = M_b$ ou aucune équation peut être dérivée. Si toutes les valeurs de paramètres valides sont prises en compte, alors la même dérivation donne également $\Delta mc^2 \propto L$ ou $L = A\Delta mc^2$, où A est un coefficient de proportionnalité. Ainsi, la dérivation d'Einstein sous des paramètres valides prédit également que l'énergie émise peut être inférieure ou supérieure à $\Delta L = \Delta mc^2$.

Key words: Einstein; Inter-Conversion of Mass and Energy; $\Delta L = \Delta mc^2$; September; 1905 Derivation; Limitation of $\Delta L = \Delta mc^2$ derivation.

I. DESCRIPTION AND CRITICAL ANALYSIS OF EINSTEIN'S THOUGHT EXPERIMENT

In Einstein's derivation the basic equation is

$$\ell^* = \ell \frac{\left[1 - \frac{v}{c}\cos\phi\right]}{\sqrt{1 - \frac{v^2}{c^2}}}, \tag{1}$$

where ℓ is the light energy emitted by a body in frame (x, y, z) and ℓ^* is the light energy measured in system (ξ, η, ζ), and v is the velocity with which the frame or system (ξ, η, ζ) is moving. This is the equation for the Doppler principle for light for any velocity whatever.[1] Equation (1) is based upon the constancy of the speed of light and c is the maximum limit for the speed of any particle. *Einstein's perception*: Let a system of plane waves of light, referred to the system of coordinates (x, y, z), possesses the energy l; let the direction of the ray (the wave-normal) make an angle φ with the axis of x of the system.[2,3] Energy is a scalar quantity, having magnitude only, but according to Eq. (1) it depends also upon angle.

Introduce a new system of coordinates (ξ, η, ζ) moving in uniform parallel translation with respect to the system (x, y, z), and having its origin of coordinates in motion along the axis of x with the velocity v. Thus, v is the relative

a)ajay.pqrs@gmail.com

 © 2013 Physics Essays Publication

TABLE I. Energies emitted before and after by the body in Einstein's September 1905 derivation.

No.	System (x, y, z) at rest	System (ξ, η, ζ) moving with velocity v
1	Before emission E_0	Before emission H_0
2	After emission E_1	After emission H_1

velocity between system (x, y, z) and system (ξ, η, ζ). The body which emits light energy is considered stationary in the system (x, y, z) and also remains stationary after emission of light energy in the system (x, y, z).

Let E_0 and H_0 be the energies in the coordinate system (x, y, z) and system (ξ, η, ζ), respectively, before emission of light energy. Furthermore, E_1 and H_1 are the energies of the body in both systems, respectively, after it emits light energy. E_1 and H_1 include all the energies possessed by the body in the two systems (see Table I).

Einstein supposed that a body emits two light waves, each of energy $0.5\,L$, in system (x, y, z), where total energy is E_0. Thus, Energy before Emission = Energy after emission $+0.5\,L + 0.5\,L$,

$$E_0 = E_1 + 0.5L + 0.5L = E_1 + L. \tag{2}$$

Energy of body in system (ξ, η, ζ),

$$H_0 = H_1 + 0.5\beta L\left\{\left(1 - \frac{v}{c}\cos\phi\right) + \left(1 + \frac{v}{c}\cos\phi\right)\right\}, \tag{3}$$

where

$$\beta = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \tag{4}$$

$$H_0 = H_1 + \beta L, \tag{5}$$

or

$$(H_0 - E_0) - (H_1 - E_1) = L[\beta - 1]. \tag{6}$$

Einstein maintained that

$$(H_0 - E_0) = K_0 + C = \frac{M_b v^2}{2} + C$$

$$(H_1 - E_1) = K_1 + C = \frac{M_a v^2}{2} + C.$$

Einstein defined C as an additive constant which depends on the choice of the arbitrary additive constants of the energies H and E. The arbitrary additive constant C is regarded as equal in both cases. The kinetic energy of the body before emission of light energy is

K_0 $(\frac{M_b v^2}{2})$ and kinetic energy of the body after emission of light energy is K_1 $(\frac{M_a v^2}{2})$,

$$K_0 - K_1 = L\left\{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1\right\}. \tag{7}$$

Einstein considered the velocity in the classical region and applied the binomial theorem,

$$K_0 - K_1 = L\left(1 + \frac{v^2}{2c^2} + 3\frac{v^4}{8c^4} + \cdots - 1\right). \tag{8}$$

Further Einstein cited,[3]

…Neglecting magnitudes of the fourth (v^4/c^4) and higher orders $(v^6/c^6, v^8/c^8\ldots)$, we may write

$$K_0 - K_1 = L\frac{v^2}{2c^2}, \tag{9}$$

$$\frac{M_b v^2}{2} - \frac{M_a v^2}{2} = L\frac{v^2}{2c^2} \tag{10}$$

or

$$L = (M_b - M_a)c^2 = \Delta mc^2 \tag{11}$$

or

$$\text{Mass of body after emission } (M_a) = \text{Mass of body before emission } (M_b) - \frac{L}{c^2}\ldots \tag{12}$$

Then Einstein generalized the result to all energy and called the mass of a body the measure of energy content (all energy that is included in a collection). Fadner[4] has mentioned that in his paper Einstein neither wrote $E = \Delta mc^2$ nor E. Hetcht[5,6] maintained that Einstein never derived his mass–energy equation scientifically. It is thus generally concluded that Einstein's statement means $E = \Delta mc^2$. However, it is merely speculation. It can be obtained by replacing L (light energy) by E (energy-content or all energy). Einstein wrote

$$E = (M_b - M_a)c^2 = \Delta mc^2 \tag{13}$$

or

$$\text{Mass of body after emission } (M_a) = \text{Mass of body before emission } (M_b) - \frac{E}{c^2}. \tag{14}$$

When energy is emitted, the mass decreases. Thus, Einstein did not differentiate between Light Energy and other forms of energy in the derivation. Δmc^2 is a measure of kinetic energy. Einstein's speculation produced his erroneous conclusion that the total energy of a body is also given by the quantity Δmc^2.

A. General comments regarding the classical region of velocity (not given by Einstein)

Einstein's derivation also produces the most mysterious situation in science. This is explored below, with the help of the relativistic variation of mass equation,[7–14]

$$M_{\text{motion}} = \frac{M_{\text{rest}}}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{15}$$

TABLE II. Terms neglected in calculations, and their effects.

No.	Velocity	$M_{rel} = M_{rest}[1 + v^2/2c^2 + 3v^4/8v^4 + \cdots]$	Neglected term	Result
1	0	$M_{rel} = M_{rest}$	None	$M_{rel} = M_{rest}$
2	Earth's orbital velocity 30 km/s or 3×10^4 m/s	$M_{rel} = M_{rest}\,[1 + 5 \times 10^{-9} + 3.75 \times 10^{-17} + \cdots]$	5×10^{-9}	$M_{rel} = M_{rest}$
3	$v = 1$ cm/s or 0.036 km/s	$K_b - K_a = L\,[1 + 5.55 \times 10^{-22} + 4.166 \times 10^{-42} + \cdots - 1]$ or $M_b = M_a$	5.55×10^{-22}	$M_b = M_a$ Mass before emission = Mass after emission

Let the velocity be in the classical region, i.e., 1 m/s (3.6 km/h), then there is no increase in the mass of an object moving with this velocity. The speed of an aeroplane is over 400 km/h, and no increase in mass is observed.

$$M_{\text{motion}} = M_{\text{rest}}\left[1 + v^2/2c^2 + 3v^4/8c^4 + \cdots\right] \tag{16}$$

(i) If v = 0, then,

$$M_{\text{motion}} = M_{\text{rest}}.$$

(ii) If $v = 1$ cm/s (0.036 km/h), then,

$$\begin{aligned} M_{\text{motion}} &= M_{\text{rest}}\left[1 + 5.55 \times 10^{-22} + 4.166 \times 10^{-42} + \cdots\right] \\ M_{\text{motion}} &= M_{\text{rest}} + M_{\text{rest}}\,5.55 \times 10^{-22} + M_{\text{rest}}\,4.166 \times 10^{-42} + \cdots \end{aligned} \tag{17}$$

Here, even the term 5.55×10^{-22} is regarded as negligible compared with unity, and 4.166×10^{-42} is also negligible, thus

$$M_{\text{motion}} = M_{\text{rest}}. \tag{18}$$

Thus, the term 5.55×10^{-22} can be neglected only when both masses are equal.

(iii) Similarly, the orbital velocity of the Earth is 30 km/s or 30,000 m/s, i.e., $v/c = 10^{-4}$, so

$$\begin{aligned} M_{\text{motion}} &= M_{\text{rest}}\left[1 + v^2/2c^2 + 3v^4/8c^4 + \cdots\right] \\ &= M_{\text{rest}}[1 + 5 \times 10^{-9} + 3.75 \times 10^{-17} + \cdots] \\ M_{\text{motion}} &= M_{\text{rest}} + M_{\text{rest}}\,5 \times 10^{-9} + M_{\text{rest}}\,3.75 \times 10^{-17}. \end{aligned} \tag{19}$$

The mass of Earth remains the same, i.e., 5.98×10^{24} kg. Thus, here the term $v^2/2c^2$ (5×10^{-9}) is also negligible compared with unity. If the term $v^2/2c^2$ (5×10^{-9}) is neglected, then

$$\begin{aligned} &M_{\text{motion}}[\text{mass of Earth in motion}] \\ &\quad = M_{\text{rest}}[\text{mass of Earth at rest}]. \end{aligned} \tag{20}$$

The various terms neglected compared with unity are shown in Tables II and III.

B. Appearance of c^2 in $\Delta L = \Delta mc^2$ is apparently arbitrary

Now consider the same case when the velocity is 1 cm/s or 0.036 km/h. Under this condition, Eq. (8) becomes

$$\frac{M_b v^2}{2} - \frac{M_a v^2}{2} = L[1 + 5.55 \times 10^{-22} + 4.166 \times 10^{-42} + \cdots - 1]. \tag{21}$$

TABLE III. Values of various parameters considered by Einstein and neglected by Einstein in the derivation of the light energy–mass equation $L = \Delta mc^2$.

No	Parameters	Einstein considered	Einstein neglected (no reason was given by Einstein why these parameters are neglected)
1	No. of light waves	Two light waves	One, three, four, or n waves
2	Energy of light wave	Equal 0.5L and 0.5 L each	Energies of the order of 0.500001 L and 0.499999 L are also possible. There are numerous such possibilities, which need to be investigated. Bodies can emit more than two waves. The invisible waves of energy are not taken into account
3	Angle	0° and 180°	The angles can be 0° and 180.001° or 0.9999° and 180°, etc. There are numerous such possibilities which need to be explored
4	Velocity	Classical region	The velocity can be in the relativistic region. The velocity v can also be zero, i.e., $v = 0$ $v \sim c$ mass increases
5	Velocity	Uniform in classical region	The law of interconversion of mass to energy also holds good when the velocity is variable

(i) Einstein has neglected the term $3v^4/8c^4$ and retained the term $v^2/2c^2$, and obtained the equation,

$$L = \Delta mc^2.$$

(ii) If the velocity is very small, then $v^2/2c^2$ can be neglected compared with unity. If the velocity is 1 cm/s (classical region), then $v^2/2c^2$ is 5.55×10^{-22}. Depending upon the orbital velocity of the Earth (30 km/s or 30,000 m/s, i.e., $v/c = 10^{-4}$), the term $v^2/2c^2$ (5×10^{-9}) can be neglected compared with unity; only then the equation,

$$M_{\text{motion}}[\text{mass of earth in motion}] = M_{\text{rest}}[\text{mass of earth at rest}]$$

is justified. In the typical classical region ($v = 1$ cm/s) $v^2/2c^2 = 5.55 \times 10^{-22}$ is neglected compared with unity (as 5×10^{-9} is neglected), then

$$M_{\text{b}}(\text{mass before emission}) = M_{\text{a}}(\text{mass after emission}). \tag{22}$$

Thus, both $\Delta L = \Delta mc^2$ and $M_{\text{b}} = M_{\text{a}}$ are equally probable but have an entirely different nature. This discussion also validates the necessity of categorization of subranges of velocity in the classical region or up to which the magnitude of the terms are to be neglected, comparatively.

II. EINSTEIN TOOK ONLY VERY SPECIAL VALUES OF VARIABLES AND THEIR EFFECTS

The following arguments show that Einstein's derivation is true under special conditions.[15–19] Einstein[3] has placed conditions on the state of the body: Let there be a *stationary body* in the system (x, y, z), and let its energy referred to the system (x, y, z) be E_0. Let the energy of the body relative to the system (ξ, η, ζ) moving as above with the velocity v, be H_0. The body also remains stationary in the system (x, y, z) after emission of energy.

Einstein also assumed that the body also remains stationary before and after emission of light energy, which is a very special condition.

But practically this condition (light emitting body is stationary) is not obeyed in many other cases.

(i) Nuclear fission is caused by thermal neutrons which have a velocity of 2185 m/s. The uranium atom also moves as it is split up in barium and krypton, and emits energy.

(ii) When a gamma ray photon of energy at least 1.02 MeV moves near the field of a nucleus it is split up into an electron and positron pair.[22] The gamma ray photon is in motion and so is the electron and positron pair.

(iii) Similarly, particle and antiparticle move towards each other for annihilation. The particle and antiparticle collide then annihilation takes place. In nuclear fusion, the atoms are set in motion. Fission is only caused by thermal neutrons (0.025 eV or having velocity 2185 m/s). Thus, there are characteristic or inherent conditions in the process of interconversion of mass and energy. These phenomena were not known in Einstein's time.

(iv) When a piece of paper burns, it is also set in motion and energy in various forms which is emitted.

(v) Deuterium and tritium fuse, but only after they are set in motion under conditions of high temperature. In the nuclear fusion of deuterium and tritium, the energy of emitted neutrons is 14.1 MeV (moving at 52,000 km/s) and their mass must increase about 15.36%. It may increase the mass considerably. The velocity of the reactants is not necessarily uniform and gradually they overcome the force of electrostatic repulsion. Chemical reactions were discovered in Einstein's time. Einstein never discussed this phenomenon in his works.

Thus, derivation subject to the condition that a body remains stationary in the emission process is not conceptually useful or applicable in other cases. In fact Einstein had in his mind the emission of radiation by radioactive sources,[20,21] which remains at rest when radiation is emitted. That the body remains stationary after emission of light energy is only a theoretical perception.

III. OTHER CONDITIONS ON EINSTEIN'S DERIVATION

Einstein's September 1905 derivation[3] of $\Delta L = \Delta mc^2$ is true only under *very special conditions or handpicked conditions*. It is justified as follows: In the derivation of $\Delta L = \Delta mc^2$, there are FOUR variables, e.g.,

(a) Number of waves emitted;
(b) l, magnitude of light energy;
(c) Angle ϕ at which light energy is emitted; and
(d) Uniform relative velocity v. Fast neutrons are slowed down and called thermal neutrons, thus their velocities are not necessarily uniform as they can be variable while they cause fission of other nuclei.

IV. NATURE OF *V* ACCORDING TO EINSTEIN

The quantity v is the relative velocity between system (x, y, z) and system (ξ, η, ζ). If system (x, y, z) is at rest and system (ξ, η, ζ) moves with constant velocity v, then v is the relative velocity. If the system (x, y, z) and system (ξ, η, ζ) both move with same velocity then the relative velocity v is zero. Furthermore, Einstein strictly took the value of velocity as uniform. The law of interconversion of mass and energy holds good if,

(i) Velocity v is in the classical region;
(ii) Velocity v is in the relativistic region;
(iii) Velocity v is zero, i.e., if both systems move with same velocity or system (ξ, η, ζ) is at rest.
(iv) Velocity v is variable or uniform.

These variables have numerous values. The law of interconversion of mass and energy holds under all conditions, but Einstein has considered just one, i.e., velocity is precisely uniform in the classical region. It does not hold under relativistic conditions. Such a significant derivation must be independent of velocity. If the binomial theorem is not applied, i.e., all values of velocity are considered then, Eq. (7) becomes $L = \Delta m v^2/2(\beta - 1)$, which is dependent on velocity.

V. GENUINE CASES NEGLECTED IN EINSTEIN'S DERIVATION

Einstein has taken very special values of parameters. Thus for a complete analysis, the derivation is herein repeated with all possible values of parameters. In all cases, the law of conservation of momentum is obeyed (which is discussed in Section VI).

(i) The body can emit a large number of light waves but Einstein has taken only *two* light waves emitted by his luminous body. Why are one or *n* light energy waves neglected?

(ii) The energy of two emitted light waves may have different magnitudes but Einstein has taken two light waves of equal magnitudes (0.5L each). Why are other magnitudes (e.g., 0.500001 L and 0.499999 L) neglected by Einstein?

(iii) A body may emit a large number of light waves of different magnitudes of energy, making different angles (other than 0° and 180° as assumed by Einstein). Why are other angles (such as 0° and 180.001°, 0.9999° and 180°, etc.) neglected by Einstein? Thus, a body has been specially fabricated; other forms of energy such as invisible energy are not taken into account. Furthermore, a body is limited to emission of light energy only, not other forms of energy.

(iv) Einstein has used velocity in the classical region ($v \ll c$) and applied the binomial theorem at the end and has not at all used velocity in the relativistic region. If velocity is considered in the relativistic region (v is comparable with c), then the equation for relativistic variation of mass with velocity, i.e., Eq. (15) is taken into account[5]. It must be noted that before Einstein's work this equation was adduced by Lorentz[8,9] and first confirmed by Kaufman[10] and afterwards more convincingly by Bucherer.[11] Einstein on 19 June, 1948 wrote a letter to Lincoln Barnett[12] and advocated abandoning relativistic mass and suggested that it is better to use the expression for the momentum and energy of a body in motion, instead of relativistic mass.

It is a strange suggestion, as Einstein used relativistic mass in his work, including in his expression for relativistic kinetic energy[1] from which rest-mass energy is derived.[22,23]

So Einstein's equation for the interconversion of mass and energy highly depends upon theoretical velocity whereas practically the mass–energy interconversion phenomena are applicable in all cases.

(v) Einstein has considered that a body emits light energy, but simultaneously the body may also emit heat energy which is not taken in account in Einstein's derivation. A burning body emits heat, sound, and light energy; energy in the form of invisible radiations simultaneously with visible radiation. For a proper description of heat energy–mass interconversion, we need an equation equivalent to Eq. (1). Similar is the case for other energies. In nuclear explosions, energies exist in many forms, e.g., light energy, sound energy, heat energy, energy in the form of various invisible radiations. Einstein has only considered Light Energy, neglecting other energies in his derivation of $L = \Delta mc^2$. Furthermore, Einstein only considered that a body emits electromagnetic energy in visible region. But energy can also be emitted in the invisible electromagnetic spectrum, but Einstein did not mention anything about heat and sound energies (emitted along with light energy). Thus energies other than light energy are also emitted but neglected by Einstein in the derivation. Einstein's derivation does not give any equation between light energy emitted and mass annihilated. So energies are not taken in account completely. Thus, Einstein's perception may be ideally regarded as a thought experiment. In a realistic scientific approach, all the salient factors have to be taken into account to draw conclusions.

VI. CONCLUSIONS

Einstein has taken only very special values of parameters and neglected many realistic values.

VII. $\Delta L \propto \Delta MC^2$ OR $\Delta L = A\,\Delta MC^2$ IS EQUALLY FEASIBLE

Consider the case in which a luminous body emits two light waves of energy 0.499999 L and 0.500001 L, respectively, in the system (x, y, z) in opposite directions.[16,17] If the waves are emitted evenly, then the body remains at rest as the velocity in this case is imperceptible, i.e., 5.3×10^{-32} m/s; which can be justified on the basis of the law of conservation of momentum.[16] Then the amount of light energy measured in both systems are related by (equivalent to the case of Einstein),[6]

$$E_0 = E_1 + L, \tag{23}$$

$$H_o = H_1 + 0.499999\,\beta L[(1 - v/c\cos 0°)] + 0.500001\,\beta L\, 1 - v/c\cos 180°)], \tag{24}$$

$$(H_o - E_o) = (H_1 - E_1) + \beta L + 0.000002\,\beta L\, v/c - L, \tag{25}$$

$$(H_o - E_o) - (H_1 - E_1) = \beta L + 0.000002\,\beta L\, v/c - L, \tag{26}$$

$$= L\left[1 + 0.000002\, v/c + v^2/2c^2 - 1\right], \tag{27}$$

$$K_b - K_a = L\left[1 + 0.000002\, v/c + v^2/2c^2 - 1\right], \tag{28}$$

$$M_b v^2/2 - M_a v^2/2 = L\left[0.000002\, v/c + v^2/2c^2\right], \tag{29}$$

$$M_b - M_a = L\left[0.000004/cv + 1/c^2\right], \tag{30}$$

$$\Delta mc^2 = L[0.000004c/v + 1], \tag{31}$$

$$L = \Delta mc^2/[0.000004c/v + 1], \tag{32}$$

$$L = \Delta mc^2/[0.000004c/v + 1] = \Delta mc^2/[1200 + 1]$$
$$= \Delta mc^2/[1201]$$
$$L \propto \Delta mc^2 \quad \text{or} \quad L = A\Delta mc^2. \tag{33}$$

So Einstein's derivation does not give a fixed value of energy corresponding to mass annihilated. Thus, if Einstein's derivation is critically analyzed, then the general result is

$$L \propto \Delta mc^2 \quad \text{or} \quad L = A\,\Delta mc^2, \tag{34}$$

where A is a coefficient of proportionality. Thus energy emitted may vary. Bakhoum[24,25] has shown that the measured yield from nuclear fission was found to be substantially less than the theoretical yield ($Q = \Delta mc^2$). Thus the energy emitted is proportional to Δmc^2.

VIII. DECREASE IN MASS IS MORE THAN L/C^2

$$M_b - M_a = L\left[0.000004/cv + 1/c^2\right], \tag{35}$$

$$M_a = M_b - 0.000004L/cv - L/c^2. \tag{36}$$

Thus, Einstein's derivation gives self-contradictory results.

Whether the effects of recoil velocity are incorporated or not, the result remains the same as velocity of recoil is 5×10^{-33} m/s.[16] There are numerous cases when the result is $L \propto \Delta mc^2$ or $L = A\ \Delta mc^2$ or $E = A\ \Delta mc^2$.

IX. LIGHT ENERGY HAS DIFFERENT INHERENT CHARACTERISTICS THAN OTHER ENERGIES

In his derivation, Einstein used Eq. (1) for relativistic variation of light energy, which was speculated in the previous papers.[15–19] But this equation is only meant for light energy not at all for other energies; hence any deduction from it must be applicable for light energy only.

Equation (1) is not meant for:

(i) Sound energy: In the Doppler effect, a change in frequency of sound is involved, not variation in mass. Thus, Eq. (1) is not used. Likewise Eq. (1) is not associated with any other energy. The speed of sound is 332 m/s at Standard Atmospheric Conditions.
(ii) Heat energy: There is no equation like Eq. (1) which relates variation of heat energy. It is similar the case of other types of energies, such as
(iii) chemical energy,
(iv) nuclear energy,
(v) magnetic energy,
(vi) electrical energy,
(vii) energy emitted in the form of invisible radiations,
(viii) attractive binding energy of nucleus,
(ix) energy emitted in cosmological and astrophysical phenomena,
(x) energy emitted in volcanic eruptions,
(xi) energies coexisting in various forms, etc.

Then why are results based upon Eq. (1) applied to the above energies (i–xi)? All energies have a different nature, and the energies are not confirmed to obey the same equation. Einstein initially derived "light energy"–mass interconversion with the equation $L = \Delta mc^2$, then speculated the "every energy"–mass interconversion equation $E = \Delta mc^2$ from his equation $L = \Delta mc^2$, when Eq. (1) is only meant for light energy, not for other energies. Hence the speculative transition to $E = \Delta mc^2$ from $L = \Delta mc^2$ is entirely without any valid mathematical basis. Einstein derivation is applicable to visible energy only not to invisible energy.

X. IF THE MEASURING SYSTEM IS AT REST ($V = 0$) AND A BODY EMITS TWO LIGHT WAVES THEN EINSTEIN'S DERIVATION IS NOT APPLICABLE

However in this case, experimentally, when light energy is emitted mass decreases. This is a serious limitation of Einstein's derivation.

When the measuring system (ξ, η, ζ) is at rest $v = 0$ then,

$$\ell^* = \ell, \tag{37}$$

$$H_o = H_1 + L/2 + L/2, \tag{38}$$

$$E_o = E_1 + L$$

$$(H_o - E_o) - (H_1 - E_1) = 0. \tag{39}$$

As a body is at rest and the measuring system (ξ, η, ζ) is also at rest, then ($H_o - E_o$) or ($H_1 - E_1$) cannot be interpreted as kinetic energy. Hence a further derivation is not applicable. This is a critical mathematical analysis, which has no implications on the experimentally established status of $\Delta L = \Delta mc^2$ ($E = \Delta mc^2$). Einstein did not let these issues stop his September 1905 paper nor his subsequent publications.[26] According to Ives,[27] the derivation Einstein attempted for the formula $E = mc^2$ was fatally flawed because Einstein set out to prove what he had already assumed.

$E = \Delta mc^2$ has been derived by using only the theory of electromagnetism. There is no other derivation for the equation, which does not rely on the theory of electromagnetism. Indeed Einstein took a very special case (the energy associated with electromagnetic energy) and made a general (and erroneous!) conclusion about mass–energy equivalence.

XI. SUPERLUMINAL SPEED OF NEUTRINOS

In addition, experiments are being conducted to measure the velocity (may be exceeding c) of neutrinos in the MIMOS+ experiments at FermiLab. The experiments aim to measure the time a neutrino takes to travel from FermiLab to the Soudan mine in Minnesota with a precision of about 1 ns.[28] So they are far more precise experiments than those conducted in 2007 by the MINOS project. If the speed of neutrinos is found even equal to c, this discussion will be further validated.

ACKNOWLEDGMENTS

The author is highly indebted to Professor Dr Shashi Kant, Stephen J. Crothers, and Anjana Sharma for critical discussions and guidance.

[1]A. Einstein, Ann. Phys. **17**, 891 (1905), http://www.fourmilab.ch/etexts/einstein/specrel/www/
[2]"The collected papers of Albert Einstein," in *The Swiss Years: Writings, 1900–1909*, translated by Anna Beck (Princeton University Press, Princeton, 1989), Vol. 2, pp. 140–171.
[3]A. Einstein, Ann. Phys. **18**, 639 (1905), http://www.fourmilab.ch/etexts/einstein/E_mc2/www/
[4]W. L. Fadner, Am. J. Phys. **56**, 144 (1988).
[5]E. Hetcht, Phys. Teach. **50**, 91 (2012).
[6]E. Hecht, Am. J. Phys. **77**, 799 (2009).
[7]J. J. Thomson, Philos. Mag. **11**, 229 (1881).
[8]H. A. Lorentz, Proc. R. Soc. Amst. **1**, 427 (1899).
[9]H. A. Lorentz, Proc. R. Soc. Amst. **6**, 809 (1904).
[10]W. Kaufmann, Phys. Z. **4**, 55 (1902); Gott. Nacchr. Math. Phys. **4**, 90 (1903).
[11]A. H. Bucherer, Phys. Z. **9**, 755 (1908); Ann. Phys. **28**, 513 (1909).
[12]A. Einstein, 1948 Letter to Lincoln Barnett, quoted in "The concept of mass" by L. Okum, Physics Today, June 1989.
[13]G. Neumann, Ann. Phys. **45**, 529 (1914).
[14]M. Jammer, *Concepts of Mass in Classical and Modern Physics* (Dover, New York, 1997).
[15]A. Sharma, in *Proceedings of Physical Science of 98th Indian Science Congress*, (Indian Science Congress Association, Chennai, 2011), p. 194.
[16]A. Sharma, Prog. Phys. **3**, 7 (2008).
[17]A. Sharma, Phys. Essays **17**, 195 (2004).
[18]A. Sharma, in *Proceedings of International Conference on Computational Methods in Sciences and Engineering* 2003 (World Scientific Co. USA, 2003), pp. 585–586.
[19]A. Sharma, in *Proceedings of International Conference on Number, Time, Relativity* (United Physical Society of Russian Federation, Moscow, 2004), p. 81.
[20]S. Frederick, *Radio-Activity: An Elementary Treatise, from the Standpoint of the Disintegration Theory* (Van Nostrand, New York, 1904), p. 164.
[21]A. Pais, *Subtle is the Lord* (Oxford University Pres., Oxford, 1982), p. 149.
[22]A. Beiser, *Concepts of Modern Physics*, 4th ed. (McGraw-Hill International Edition, New York, 1987), pp. 25, 27, 420, 428.
[23]R. Resnick, *Introduction to Special Theory of Relativity* (John Wiley and Sons, New York, 1968), pp. 120–121.
[24]E. Bakhoum, Phys. Essays **15**, 87 (2002).
[25]E. Bakhoum, Phys. Essays **15**, 439 (2002)
[26]A. Einstein, Bull. Am. Math. Soc. **41**, 223 (1935).
[27]H. E. Ives, J. Opt. Soc. Am. **42**, 540 (1952).
[28]K. Riesselmann, Scientific Communications, 5 July 2013, kurtr@fnal.gov.